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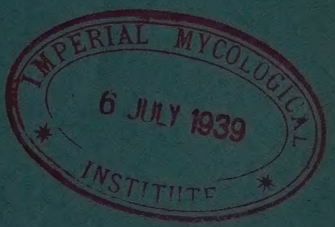

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MATHÉMATIQUES

SUR UN SYSTÈME D'ÉQUATIONS DE DIOPHANTE

Par CONSTANTIN MARDJANICHVILI

(Présenté par I. M. Vinogradow, de l'Académie, le 28. I. 1939)

Considérons le système d'équations de Diophante:

$$\left.\begin{aligned} x_1 + x_2 + \ldots + x_s &= N_1 \\ x_1^2 + x_2^2 + \ldots + x_s^2 &= N_2 \\ \ldots\ldots\ldots\ldots\ldots & \\ x_1^n + x_2^n + \ldots + x_s^n &= N_n \end{aligned}\right\} \qquad (1)$$

Dans notre article «Sur la représentation simultanée de n nombres par des sommes de puissances complètes» (1) nous avons donné une formule asymptotique pour le nombre $I(N_1, \ldots, N_n)$ des solutions du système (1) en nombres entiers $x_1, \ldots, x_s$ non-négatifs pour le cas $s \geqslant 2^{2n} n!\,(n+1)^3$. Dans le présent article, en nous appuyant sur un lemme de I. M. Vinogradow, nous démontrons qu'on peut encore établir cette formule pour le cas où s est essentiellement plus petit.

À savoir, nous démontrons le théorème suivant:

Soit n un entier $\geqslant 2$, s — un entier $\geqslant 5n(n+1)(n+2)\lg n$. Alors on peut choisir les nombres réels $E_k = E_k(n,s) > 1$; $e_k = e_k(n,s) < 1$ de telle manière, que pour des entiers positifs quelconques $N_1, N_2, \ldots, N_n$, satisfaisant aux conditions

$$\left.\begin{aligned} N_k &= h_k N_n^{\frac{k}{n}} \\ E_k &\leqslant h_k \leqslant s^{1-\frac{k}{n}} e_k \end{aligned}\right\} \quad (k = 1, 2, \ldots, n-1) \qquad (2)$$

le nombre des solutions du système (1) *en nombres entiers non-négatifs s'exprime par la formule:*

$$I(N_1, \ldots, N_n) = B_1(n, s; h_1, \ldots, h_{n-1}) N_n^{\frac{s}{n} - \frac{n+1}{2}} \times \\ \times [\mathfrak{S}(N_1, \ldots, N_n) + O(N_n^{-\omega})], \qquad (3)$$

où

$$\omega = \frac{1}{2^{n+4} n!\, n^2},$$

$$B_1(n, s; h_1, \ldots, h_{n-1}) \geqslant B_0(n, s) > 0,$$

$$\mathfrak{S}(N_1, \ldots, N_n) = \sum_{q_1, \ldots, q_n = 1}^{\infty} A(q_1, \ldots, q_n), \qquad (4)$$

$$A(q_1, \ldots, q_n) = \sum_{a_1, \ldots, a_n}{}' D^s e^{-2\pi i \left(\frac{a_1}{q_1} N_1 + \ldots + \frac{a_n}{q_n} N_n\right)} \qquad (5)$$

($a_1, \ldots, a_n$ *parcourent des systèmes réduits de résidus resp. modulo* $q_1, \ldots, q_n$) *et*

$$D = \frac{1}{q_1 \cdots q_n} \sum_{1 \leqslant x \leqslant q_1 q_2 \cdots q_n} e^{2\pi i \left(\frac{a_n}{q_n} x^n + \ldots + \frac{a_1}{q_1} x\right)}. \tag{6}$$

Démonstration. 1. Soient $N_1 < N_2 < \ldots < N_n$ des entiers positifs donnés. Posons

$$N_n = P^n$$
$$N_k = h_k P^k \quad (k = 1, 2, \ldots, n-1).$$

Supposons que pour les entiers positifs $M_1, \ldots, M_n$ on a

$$N_r - N_r P^{-\delta} \leqslant M_r \leqslant N_r,$$

où

$$\delta = \frac{1}{8n}.$$

Le nombre $I(M_1, \ldots, M_n)$ des solutions du système

$$x_1^r + x_2^r + \ldots + x_s^r \quad (r = 1, 2, \ldots, n) \tag{7}$$

se représente sous la forme

$$I(M_1, \ldots, M_n) = \int_0^1 \cdots \int_0^1 T^s e^{-2\pi i(\alpha_n M_n + \ldots + \alpha_1 M_1)} d\alpha_n \ldots d\alpha_1, \tag{8}$$

où

$$T = \sum_{0 \leqslant x \leqslant P} e^{2\pi i(\alpha_n x^n + \ldots + \alpha_1 x)}. \tag{9}$$

2. Posons

$$\lambda_k = \frac{1}{2^{k+2} k! n} \quad (k = 1, 2, \ldots, n).$$
$$\tau_k = P^{k - \lambda_k}$$

Représentons chaque α_k entre 0 et 1 sous la forme

$$\alpha_k = \frac{a_k}{q_k} + z_k,$$

où

$$(a_k, q_k) = 1, \quad 1 \leqslant q_k \leqslant \tau_k, \quad |z_k| \leqslant \frac{1}{q_k \tau_k}. \tag{10}$$

Pour obtenir la partie principale de $I(M_1, \ldots, M_n)$, considérons les $\alpha_1, \ldots, \alpha_n$ dont les $q_1, \ldots, q_n$ correspondants satisfont aux conditions

$$q_k \leqslant P^{\lambda_k} * \quad (k = 1, 2, \ldots, n).$$

En reproduisant la démonstration du § 3 de la première partie de mon ouvrage (1), nous trouverons pour la somme $I_1(M_1, \ldots, M_n)$ des intégrales étendues aux paralléloïdes

$$\frac{a_k}{q_k} - \frac{1}{\tau_k} \leqslant z_k \leqslant \frac{a_k}{q_k} + \frac{1}{\tau_k}, \tag{11}$$

---

* $A \ll B$ a le même sens que $A = O(B)$.

considérée pour toutes les valeurs de $q_1, a_1; \ldots; q_n, a_n$, satisfaisant aux conditions (10) et (11), que

$$I_1(M_1, \ldots, M_n) = Q \sum_{1 \leqslant q_1 \leqslant P^{\lambda_1}} \cdots \sum_{1 \leqslant q_n \leqslant P^{\lambda_n}} A(q_1, \ldots, q_n) + O\left(P^{s - \frac{n(n+1)}{2} - \frac{1}{4n}}\right),$$

où

$$Q = \int_{-\frac{1}{\tau_1}}^{\frac{1}{\tau_1}} \cdots \int_{-\frac{1}{\tau_1}}^{\frac{1}{\tau_1}} \rho^s e^{-2\pi i \sum_{k=1}^{n} z_k N_k} \, dz_n \ldots dz_1$$

avec

$$\rho = \int_0^P e^{2\pi i \sum_{k=1}^{n} z_k t^k} \, dt.$$

Nous allons montrer dans la suite que la partie restante de l'intégrale I_2 sera

$$I_2 \ll P^{s - \frac{n(n+1)}{2} - n\omega}.$$

3. Considérons à présent les $\alpha_1, \ldots, \alpha_n$ dont les $q_1, \ldots, q_n$ correspondants sont tels, que pour l'un d'eux au moins q_r on a l'inégalité

$$q_r > P^{r - \lambda_r}.$$

Si s est impair, on a

$$|I_2| < (\max |T|) \int_0^1 \cdots \int_0^1 |T|^{s-1} d\alpha_n \ldots d\alpha_1,$$

$$|I_2| < (\max |T|) \int_0^1 \cdots$$

$$\ldots \int_0^1 e^{2\pi i \alpha_n (x_1^n + \ldots + x_{\frac{s-1}{2}}^n - x_{\frac{s+1}{2}}^n - \ldots - x_{s-1}) + \ldots + 2\pi i \alpha_1 (x_1 + \ldots + x_{\frac{s-1}{2}} - x_{\frac{s+1}{2}} - \ldots - x_{s-1})} d\alpha_n \ldots d\alpha_1.$$

En estimant $|T|$ comme dans les §§ 4 et 5 de l'ouvrage (1) et en observant que le nombre des solutions du système

$$\left.\begin{aligned} x_1^n + \ldots + x_{\frac{s-1}{2}}^n - x_{\frac{s+1}{2}}^n - \ldots - x_{s-1}^n = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ x_1 + \ldots + x_{\frac{s-1}{2}} - x_{\frac{s+1}{2}} - \ldots - x_{s-1} = 0 \end{aligned}\right\},$$

satisfaisant aux conditions $0 \leqslant x_i \leqslant P$ pour la valeur choisie de s sera en vertu du lemme (6) de l'ouvrage (2) de I. M. Vinogradow

$$\ll P^{s-1-\frac{n(n+1)}{2}},$$

nous trouverons

$$I_2 \ll P^{s - \frac{n(n+1)}{2} - n\omega}.$$

Si s est pair, nous recevons le même résultat en mettant $(\max |T|)^2$ hors du signe de l'intégrale.

4. En somme pour le nombre $I(M_1, \ldots, M_n)$ des solutions du système (7) nous obtenons:

$$I(M_1, \ldots, M_n) = Q \sum_{1 \leqslant q_1 \leqslant P^{\lambda_1}} \cdots \sum_{1 \leqslant q_n \leqslant P^{\lambda_n}} A(q_1, \ldots, q_n) + O\left(P^{s - \frac{n(n+1)}{2} - n\omega}\right)$$

d'où il suit que

$$I(M_1, \ldots, M_n) = Q\mathfrak{S} + O\left(P^{s - \frac{n(n+1)}{2} - n\omega}\right).$$

En calculant Q comme dans les §§ 7 et 8 de mon article [1] nous avons:

$$I(N_1, \ldots, N_n) = B_1(n, s; h_1, \ldots, h_{n-1}) P^{s - \frac{n(n+1)}{2}} [\mathfrak{S}(N_1, \ldots, N_n) + O(P^{-n\omega})],$$

où

$$B_1(n, s; h_1, \ldots, h_{n-1}) \geqslant B_0(n, s) > 0,$$

ce qui démontre le théorème.

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MATHEMATICS

LINEAR TOPOLOGICAL SPACES AND THEIR CONNEXION WITH THE BANACH SPACES

By V. ŠMULIAN

(Communicated by I. M. Vinogradow, Member of the Academy, 28. I. 1939)

I. Let E_T denote a linear topological space, i. e. a linear space which is topological and in which the operations of addition and multiplication by a scalar are continuous. This definition belongs to A. Kolmogoroff (1). An other definition for a linear topological space was proposed by J. Neumann (2). J. Neumann's definition is based on the consideration of neighbourhoods which satisfy some 6 axioms. J. Neumann showed that the linear topological space in this sense is also a linear topological space in the sense of A. Kolmogoroff. V. Wehausen showed (3) that, conversely, every linear topological space in the sense of A. Kolmogoroff satisfies all 6 axioms of J. Neumann except the second.

Since, however, J. Neumann uses his second axiom only for proving the theorems 16—20 of his paper, in studying the linear topological space in the sense of A. Kolmogoroff, we can make use of the principal results of J. Neumann. Besides, Wehausen remarked (3) that locally convex linear topological spaces of Kolmogoroff coincide with convex linear topological spaces of J. Neumann.

Theorem 1. *A compact set of a linear topological space is totally bounded* (4).

This theorem, the proof of which is not difficult, contains a theorem of D. H. Hyers (5).

Theorem 2. *Let E_T denote a topologically complete* (6) *locally convex linear topological space. If S is a compact set in E_T, the smallest convex set containing S is also compact.*

To prove this theorem it is sufficient to use the preceding theorem and the theorems 14 and 11 of J. Neumann.

Definition 1. The set $S \subset E_T$ will be said to be complete, if for every fundamental [in the sense of G. Birkhoff (7)] sequence $\{x_\alpha\} \subset S$, where α runs over some directed set, there exists an element $x_0 \in E$, such that [in the sense of G. Birkhoff (7)] $x_\alpha \to x_0$.

Theorem 3. *A closed set $S \subset E_T$ is bicompact, if and only if it is totally bounded and complete.*

Proof. If S is a bicompact set, its completeness follows from the definition. By theorem 1 S is totally bounded. The second part of the theorem follows easily from the lemma of G. Birkhoff at p. 49 of his

paper referred to above. We note that theorem 3 is a generalization of G. Birkhoff's theorem 12.

Definition 2. A space E_T will be said to be quasi-complete, if every its totally bounded set is complete.

From this definition follows that E_T is topologically complete, if it is quasi-complete. The following theorem shows when the converse is true.

Theorem 4. *If a topological linear space of J. Neumann is topologically complete, it is also quasi-complete.*

To prove this theorem it is necessary to use the theorem 16 of J. Neumann and the preceding theorem.

Theorem 5. *Let K be a complete, convex and closed set in a locally convex linear topological space E_T. If $U(x)$ is a continuous operation on K to its compact part, there exists an element $x \in K$ such that*

$$U(x) = x.$$

Remark. If the space is quasi-complete, the condition that K is complete can be omitted.

To prove the theorem we consider the smallest convex closed set K_1 containing $U(K)$. Since K_1 is a bicompact set and $U(K_1) \subset K_1$, it remains to use the theorem of Tychonoff on the existence of a fix point [8].

II. Let E denote a Banach space *. Then we denote by $U(f_1, \dots f_n; \varepsilon)$, where $f_i \in \overline{E}$ $(i = 1, 2, \dots, n; \varepsilon > 0)$ the set of all elements $x \in E$ for which

$$|f_1^{(x)}| < \varepsilon, \dots, |f_n^{(x)}| < \varepsilon.$$

Each set $U(f_1, \dots, f_n; \varepsilon)$ for arbitrary $f_1, \dots, f_n, \varepsilon$ we call a neighbourhood of the point θ. In this way we obtain a linear topological space (in the sense of A. Kolmogoroff) which we denote by E_T. We note that this space E_T is always locally convex.

If E is separable, E_T is a linear topological space in the sense of J. Neumann.

Lemma. *If K is a convex and closed set in E, it is also a convex and closed set in E_T.*

Theorem 6. *In order that the unit sphere of the space E should be weakly compact, it is necessary and sufficient that E_T should be topologically complete.*

Theorem 7. *In order that the unit sphere of the space E should be weakly compact, it is necessary and sufficient that it should form a compact and closed set in E_T.*

To prove these two theorems it is necessary to use our lemma, the theorem 22 of J. Neumann and the following theorem of the author**:

In order that the unit sphere E should be weakly compact, it is necessary and sufficient that for every denumerable bounded sequence of convex closed sets

$$K_1 \supset K_2 \supset \dots \supset K_n \supset \dots$$

enclosed in one another, there should exist an element x belonging to all K_n.

In proving the next theorem we shall apply a proposition which has been recently proved by H. Goldstine [10].

A) *Every $F \in \overline{\overline{E}}$ (the space conjugate to $\overline{E}$) can be represented in the following form: $F(f) = \lim f(x_\alpha)$, where $|x_\alpha| \leqslant |F|$ and α runs over some directed set.*

---

* We shall keep the denotations and the terminology of Banach [9].

** Which is to appear in the «Recueil Mathématique de Moscou».

Consequently, the regularity of E is equivalent to the following condition:

If for some $\{x_\alpha\}$ there exists $\lim f(x_\alpha)$ *for* $f \in \overline{E}$, *where* $|x_\alpha| \leqslant 1$ *and* α *runs over some directed set, then there exists such an element* $x_0 \in E$ *that*

$$\lim f(x_\alpha) = f(x_0) \text{ for } f \in \overline{E}.$$

It is easy to see that this criterion of H. Goldstine can be put in the following form:

The regularity of E is equivalent to the quasi-completeness of E_T.

For this it is sufficient to use the theorem 22 of J. Neumann. Now, by theorem 3 we have:

Theorem 8. *In order that the space E should be regular, it is necessary and sufficient that its unit sphere should form a bicompact set in E_T.*

III. The following theorem completes the theorem A) of H. Goldstine.

Theorem 9. *Let the unit sphere of E be weakly compact. Then every $F \in \overline{E}$ with $|F| = 1$ can be represented in the following form:*

$$F(f) = \lim f(x_\alpha),$$

where all x_α belong to some convex set lying on the boundary of the unit sphere E and α runs over some directed set. Therefore the regularity of E in this case is equivalent to the following condition: if for some $\{x_\alpha\}$, belonging to a convex set lying on the boundary of the unit sphere E, there exists $\lim f(x_\alpha)$ *for all $f \in \overline{E}$, where α runs over some directed set, then there exists an element $x_0 \in E$ such that* $\lim f(x_\alpha) = f(x_0)$ for $f \in \overline{E}$.

The proof of this theorem, as well as of the theorem A) of H. Goldstine, is based upon the following fact: every $F \in \overline{\overline{E}}$ can be represented in the form

$$F(f) = \int_Q f(x)\, d\Phi(e),$$

where Q is the unit sphere of E and $\Phi(e)$ is an additive function of bounded variation on the sets $e \in Q$.

By comparison with the preceding theorem we find

Theorem 10. *In order that a space E should be regular, it is necessary and sufficient that E_T should be topologically complete, and every convex closed (by norm) set lying on the boundary of the unit sphere E should form a bicompact set in E_T.*

After this paper was sent to print, the author got information that both theorem 8 and theorem 3 (for regularly topological spaces) had recently been proved by N. Bourbaki (C. R., Paris, 206, 1701) and H. Weyl (Actualités scient. et industr., Publ. Inst. Math. Univ. de Strasbourg, № 551) respectively.

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MATHÉMATIQUES

CONTRIBUTION À LA THÉORIE DES CONGRUENCES DE SPHÈRES

Par J. DUBNOV (DOUBNOFF) et M. SABYROV

(Présenté par N. N. Lusin, de l'Académie, le 21. I. 1939)

1. Le réseau de Meusnier associé à la congruence. Considérons une surface définie par deux tenseurs de Gauss:

$$G_{ij} = \mathbf{R}_i \mathbf{R}_j, \qquad B_{ij} = -\mathbf{R}_i \mathbf{N}_j, \tag{1}$$

où $\mathbf{R}(u^1, u^2)$ est le rayon-vecteur d'un point de la surface, $\mathbf{N}(u^1, u^2)$ — le vecteur unitaire de la normale, $\mathbf{R}_i = \frac{\partial \mathbf{R}}{\partial u^i}$. L'ensemble des courbes, situées sur la surface et ayant en chacun de ses points un rayon $\rho(u^1, u^2)$ donné de courbure normale, forme un réseau

$$(G_{\alpha\beta} - \rho B_{\alpha\beta})\, du^\alpha\, du^\beta = 0 \tag{2}$$

symétrique par rapport aux lignes de courbure. En même temps est définie une congruence de sphères (orientées) de rayon $\rho(u^1, u^2)$ dont $\mathbf{R}$ est une nappe de l'enveloppe et chaque sphère est la «sphère de Meusnier» pour deux courbes (2) passant par le point de contact. Inversement, il correspond à chaque congruence de sphères tangentes à la surface $\mathbf{R}$ un réseau (2) qu'il convient de nommer «réseau de Meusnier associé à la congruence donnée». Ce réseau ne dégénère pas, si ρ ne coïncide avec aucun des rayons de courbures principales de la surface $\mathbf{R}$, c'est-à-dire si

$$1 - 2\rho H + \rho^2 K \neq 0 \tag{3}$$

(K et H étant respectivement la courbure de Gauss et la courbure moyenne de la surface $\mathbf{R}$), ce que nous supposerons dans la suite; quand la condition (3) est vérifiée, la seconde nappe $\overset{\times}{\mathbf{R}}(u^1, u^2)$ de l'enveloppe diffère de la première. Signalons trois cas particuliers, suivant que le rayon ρ de la sphère est par rapport aux rayons de courbures principales de la surface $\mathbf{R}$ ou bien: 1) la moyenne arithmétique $\left[\rho = \frac{H}{K}$ — Mittenkugel chez Blaschke (1)$\right]$, ou bien 2) la moyenne harmonique $\left(\rho = \frac{1}{H}$ — Zentralkugel chez Blaschke$\right)$, ou bien 3) la moyenne proportionnelle $\left(\rho = \sqrt{\frac{1}{K}}\right)$; dans ces cas nous allons considérer respectivement les congruences de

sphères: 1) «moyennes arithmétiques», 2) «moyennes harmoniques», 3) «moyennes proportionnelles» par rapport à la surface **R**. Les réseaux associés de Meusnier sont respectivement:

1) $(KG_{\alpha\beta} - HB_{\alpha\beta})\, du^\alpha\, du^\beta = 0$, ce que l'on appelle «réseau caractéristique», c'est-à-dire le réseau conjugué et harmonique au réseau des lignes de courbure;

2) $(HG_{\alpha\beta} - B_{\alpha\beta})\, du^\alpha\, du^\beta = 0$, formé de lignes bissectrices du réseau des lignes de courbure;

3) $(\sqrt{K}G_{\alpha\beta} - B_{\alpha\beta})\, du^\alpha\, du^\beta = 0$.

2. La congruence sphérique de Ribaucour est caractérisée par la propriété qu'aux lignes de courbure sur la nappe **R** de l'enveloppe correspond un réseau conjugué sur la surface **r**, formée par les centres des sphères ou (ce qui est le même dans le cas où la seconde nappe $\overset{\times}{\mathbf{R}}$ de l'enveloppe ne dégénère pas) les nappes **R** et $\overset{\times}{\mathbf{R}}$ se correspondent par les lignes de courbure. Quand on connaît les tenseurs (1) de la nappe **R** et le rayon ρ de la sphère, on peut trouver à un facteur près le second tenseur de Gauss b_{ij} de la surface **r** des centres:

$$b_{ij} :: (1 - 2\rho H + \rho^2 K)\,[\nabla_j \rho_i + \rho K G_{ij} - (2\rho H - 1)\, B_{ij}] + \\ + \rho_\omega\,[\rho B^\omega_\alpha - (2\rho H - 1)\,\delta^\omega_\alpha]\ \ (\rho \nabla_j B^\alpha_i + \rho_i B^\alpha_j + \rho_j B^\alpha_i), \tag{4}$$

où :: est le signe de proportionalité, ∇ est le symbole de dérivation covariante effectuée par rapport au tenseur métrique G_{ij}, enfin $B^h_i = G^{h\alpha} B_{i\alpha}$. En partant de (4), on arrive à la condition suivante caractérisant la congruence de Ribaucour:

$$\left.\begin{aligned} &\xi_i \equiv \frac{(B^\alpha_i - \rho K \delta^\alpha_i)\,\rho_\alpha}{1 - 2\rho H + \rho^2 K} = \operatorname{grad} \\ &\text{ou bien} \\ &E^{\lambda\mu} \nabla_\lambda \frac{(B^\alpha_\mu - \rho K \delta^\alpha_\mu)\,\rho_\alpha}{1 - 2\rho H + \rho^2 K} = 0, \quad (E^{11} = E^{22} = 0, \quad E^{12} = -E^{21}). \end{aligned}\right\} \tag{5}$$

Ainsi la recherche de toutes les «transformations de Ribaucour» pour la surface définie en coordonnées générales par les tenseurs (1) dépend d'une équation différentielle aux dérivées partielles du 2 ordre par rapport à une fonction inconnue φ. Remarquons que les composantes du tenseur ξ_i figurant dans (5) prennent une forme simple quand on rapporte la surface **R** à ses lignes de courbure:

$$\xi_1 = \frac{\rho_1}{\rho' - \rho}, \qquad \xi_2 = \frac{\rho_2}{\rho'' - \rho},$$

ρ' et ρ'' étant les rayons de courbures principales de cette surface.

En revenant au cas général, nous trouvons le «tenseur de Tschebychev» τ_i [1] du réseau (2) associé à la congruence

$$2\tau_i = \frac{H\rho_i - \rho H_i - B^\alpha_i \rho_\alpha + \frac{1}{2}\,\rho^2 K}{1 - 2\rho H + \rho^2 K}.$$

En excluant $B^\alpha_i \rho_\alpha$ de cette expression et de l'expression (5) du tenseur ξ_i, nous obtenons:

$$4\tau_i + 2\xi_i = \frac{4\,(H - \rho K)\,\rho_i}{1 - 2\rho H + \rho^2 K} + [\ln (1 - 2\rho H + \rho^2 K)]_i \tag{6}$$

Considérons le cas où le second membre est un gradient, ou, ce qui est le même, quand la fraction $\frac{H-\rho K}{1-\rho H}$ $\left(\text{ou bien } \frac{1-\rho H}{H-\rho K}\right)$ est une fonction de ρ; remarquons que cette dernière condition est remplie pour chacune des trois congruences citées à la fin du n° 1. Il suit de (6) que dans le cas considéré les deux propriétés: 1) $\xi_i = \text{grad}$, c'est-à-dire la congruence de sphères appartient à la classe de Ribaucour, 2) $\tau_i = \text{grad}$, c'est-à-dire le réseau associé de Meusnier est un réseau de Codazzi *, sont chacune une conséquence de l'autre.

Parmi les faits géométriques qui en découlent signalons le suivant: pour qu'un réseau caractéristique ait des invariants ponctuels de Laplace-Darboux égaux, il est nécessaire et suffisant que la congruence des sphères moyennes arithmétiques soit cell de Ribaucour.

3. On obtient trois généralisations de la classe des congruences de Ribaucour en affaiblissant les restrictions contenues dans la définition de cette classe. A savoir, en correspondance avec les trois propriétés connues des lignes de courbure, nous remplacerons l'ancienne hypothèse (n° 2) par l'une des trois suivantes: qu'aux lignes de courbure sur l'une des nappes $\mathbf{R}$ de l'enveloppe il corresponde sur l'autre nappe $\tilde{\mathbf{R}}$ un réseau: 1) orthogonal, 2) conjugué, 3) ayant pour représentation sphérique un réseau orthogonal. Chacune de ces conditions conduit à la classe de Ribaucour complétée (abstraction faite de quelques cas spéciaux où la nappe $\tilde{\mathbf{R}}$ est une surface développable, ou bien minima, ou bien une sphère) respectivement par les congruences: 1) des moyennes géométriques, 2) des moyennes proportionnelles, 3) des moyennes arithmétiques des sphères de la nappe $\mathbf{R}$.

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* Dans l'article [2] on a nommé ainsi un réseau dont l'équation peut être écrite dans la forme $\varphi_{\alpha\beta}\, du^\alpha du^\beta = 0$ à tenseur symétrique $\nabla_k \varphi_{ij}$. Le réseau isothermique est un réseau de Codazzi orthogonal; le réseau des invariants ponctuels de Laplace-Darboux égaux est un réseau de Codazzi conjugué.

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MECHANICS

GENERALIZATION OF HAMILTON-JACOBI'S THEOREM TO QUASI-CO-ORDINATES

By V. V. DOBRONRAVOV

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The present paper contains a generalization of the Hamilton-Jacobi theorem to the case, when the so-called quasi-co-ordinates are introduced into the equations of motion of a mechanical system by the method of Boltzmann-Hamel. This gives us the possibility of direct application of the Hamilton-Jacobi theorem to non-holonomic systems.

The problem of application of the Hamilton-Jacobi theorem to non-holonomic systems was investigated by S. A. Tchaplyghin (1) and Quanjel (2). Both these authors used methods differing from those of the present paper and the results obtained by them are formulated in a different way as compared to ours.

S. A. Tchaplyghin considered the reduction of his equations to a canonical form, whereby it was necessary first to determine the so-called «reducing factor». Quanjel started from Appell's equations in the form

$$\frac{d}{dt}\frac{\partial T}{\partial q_i'} - \frac{\partial T}{\partial q_i} = Q_i + R_i,$$

where R_i denotes a rather complicated expression involving alternate derivatives of the constraint coefficients. For the application of Hamilton-Jacobi's theorem it was, moreover, necessary that all R_i should satisfy the condition $R_i = \frac{\partial \varphi}{\partial q_i}$, where φ is a function of the q_i. This last condition, however, is by far not always satisfied. Quanjel did not give a corresponding example.

§ 1. In 1902 Boltzmann (3) and, independently of him, Hamel in 1904 proposed equations of motion of a mechanical system valid for holonomic as well as for non-holonomic constraints.

The method of deduction of Boltzmann-Hamel' s equations consisted in introducing certain functions, which later became known in mechanics as «quasi-co-ordinates» (4). Supposing, namely, that $q_1, q_2, \ldots, q_k$ are the generalized co-ordinates of the system, consider the following linear forms in the derivatives:

$$\omega_\rho = \sum_{\mu=1}^{k} \alpha_{\rho\mu} q_\mu', \qquad (1)$$

where the $\alpha_{\rho\mu}$ are functions of the q_i and the determinant $|\alpha_{\rho\mu}| \neq 0$. Then

$$q_\mu' = \sum_{\lambda=1}^{k} \beta_{\lambda\mu} \omega_\lambda. \qquad (2)$$

Putting conventionally

$$\omega_\rho = \frac{d\pi_\rho}{dt}, \qquad (3)$$

we see that according to the integrability or non-integrability of the forms (1) some of the π_ρ will be true new co-ordinates of the system and other «quasi-co-ordinates» (non-existing).

The Boltzmann-Hamel equations will then have the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \omega_\lambda} + \sum_{s=1}^{k}\sum_{r=1}^{k} \gamma_{\lambda sr}\frac{\partial T}{\partial \omega_s}\omega_r - \left(\frac{\partial T}{\partial \pi_\lambda}\right) = \Pi_\lambda^* \quad (\lambda = 1, 2, \ldots k) \qquad (4)$$

where by $\left(\frac{\partial T}{\partial \pi_\lambda}\right)$ is denoted the expression $\sum_{\mu=1}^{k}\frac{\partial T}{\partial q_\mu}\beta_{\lambda\mu}$ and the $\Pi_\lambda = \sum_{\mu=1}^{k}\beta_{\lambda\mu}Q_\mu$ are the generalized forces. The functions $\gamma_{\lambda sr}$ are Hamel's coefficients.

In the case of non-holonomic and, broadly speaking, rheonomic constraints of the form

$$q'_{e+m} = \sum_{h=1}^{e} \alpha_{mh}\, q'_h + \alpha_m \quad (m = 1, 2, \ldots g; \quad e+g = k)$$

it is possible to introduce quasi-co-ordinates ω_ρ in the following way:

$$\omega_\nu = q'_\nu; \quad \delta\pi_\nu = \delta q_\nu \quad (\nu = 1, 2, \ldots e)$$

$$\left.\begin{aligned} \omega_{e+m+1} &= q'_{e+m+1} - \sum_{h=1}^{e} \alpha_{mh}\, q'_h - \alpha_m \\ \delta\pi_{e+m+1} &= \delta q_{e+m+1} - \sum_{h=1}^{e} \alpha_{mh}\, \delta q_h \end{aligned}\right\} \quad m = 1, 2, \ldots g.$$

In virtue of the constraint condition $\omega_{e+m+1} = 0$ the equation (4) will then take the form:

$$\frac{d}{dt}\frac{\partial T}{\partial \omega_h} + \sum_{s=1}^{e+1}\sum_{m=1}^{g} \gamma_{h,\, e+m+1,\, s}\frac{\partial T}{\partial \omega_{e+m+1}}\omega_s - \left(\frac{\partial T}{\partial \pi_h}\right) = \Pi_h \quad (h = 1, 2, \ldots e). \qquad (5)$$

The equation (5) together with the constraint equations give now a system of equations for the determination of the $q_1, q_2, \ldots, q_k$ as functions of the time t.

§ 2. We shall now show that the Hamilton-Jacobi theorem may be generalized to the case of quasi-co-ordinates, i. e. that the equations (4) and (5) may be reduced to the canonical form and that for them it is possible to find a partial differential equation. the knowledge of the complete integral of which enables us to obtain the solution of the system (4) or (5).

In order to reduce the equations (4) to the canonical form, we introduce new variables $p_\lambda = \frac{\partial T}{\partial \omega_\lambda}$. Further, we put $K = \sum_{\mu=1}^{k} p_\mu \omega_\mu - T$.

* For the particular case $\gamma_{\lambda sr} = \text{const}$ and for holonomic systems only, these equations had been obtained as early as in 1901 by H. Poincaré. who applied practically the same method of «quasi-co-ordinates» [5] by means of the S. Lie groups.

Considering the function K first as a function of the p_μ, ω_μ and q_μ and then as a function of the p_μ and q_μ only, we find the following relations:

$$\omega_\mu = \frac{\partial K}{\partial p_\mu}, \quad \left(\frac{\partial K}{\partial \pi_\mu}\right) = -\left(\frac{\partial T}{\partial \pi_\mu}\right), \tag{6}$$

where, in general,

$$\left(\frac{\partial f}{\partial \pi_\mu}\right) = \sum_{i=1}^{k} \frac{\partial f}{\partial q_i} \beta_{\mu i}.$$

The equation (4) will be then written in the form:

$$\frac{dp_\lambda}{dt} + \sum_{s=1}^{k}\sum_{r=1}^{k} \gamma_{\lambda sr} \frac{\partial T}{\partial \omega_s} \frac{\partial K}{\partial p_r} = -\left[\frac{\partial (K-U)}{\partial \pi_\lambda}\right], \tag{7}$$

putting $\Pi_\lambda = \left(\frac{\partial U}{\partial \pi_\lambda}\right)$.

We introduce now the function H: $H = K - U = \sum_{\lambda=1}^{k} p_\lambda \omega_\lambda - T - U$.

If T is homogeneous in the ω_λ, $H = T - U$, and the equation (7) will have the form:

$$\left.\begin{aligned} \omega_\lambda &= \frac{\partial H}{\partial p_\lambda}, \\ \frac{dp_\lambda}{dt} &= -\left(\frac{\partial H}{\partial \pi_\lambda}\right) - \sum_{s=1}^{k}\sum_{r=1}^{k} \gamma_{\lambda sr} p_s \frac{\partial H}{\partial p_r}. \end{aligned}\right\} \tag{8}$$

These are the canonical equations in quasi-co-ordinates.

§ 3. Let us now introduce the operator $\beta_i(\varphi)$:

$$\beta_i(\varphi) = \beta_{i1}\frac{\partial \varphi}{\partial q_1} + \beta_{i2}\frac{\partial \varphi}{\partial q_2} + \ldots + \beta_{ik}\frac{\partial \varphi}{\partial q_k}, \tag{9}$$

where the β_{ij} are the coefficients occurring in (2).

The following proposition holds. If we know the complete integral of the equation

$$\frac{\partial \varphi}{\partial t} + H[\beta_i(\varphi), \ q_i, \ t] = 0, \ (i = 1, 2, \ldots k). \tag{10}$$

i. e. if we know a function $\varphi(q_i, a_i, t)$, where $a_i = \text{const}$, satisfying the equation (10), we obtain the solution of the canonical equations in quasi-co-ordinates, i. e. of the equations (8), putting

$$\frac{\partial \varphi}{\partial a_i} = b_i; \quad p_i = \beta_i(\varphi) \ (b_i = \text{const}). \tag{11}$$

The proof is as follows*. We differentiate with respect to the time each of the equations (11) and express each q_i' through the ω_λ and the $\frac{\partial H}{\partial p_\lambda}$, using the equations (8) and replacing each $\frac{dp_i}{dt}$ also through its expression from (8). Further we show that the equations obtained by differentiating the equations (11) are identities. To this end we substitute the known complete integral into the equation (10) and with the so obtained identity perform the following operations: first we differentiate with respect to the a_i and obtain an identity coinciding with the equation, which we obtain by differentiating the first of the equations (11) and which, consequently, is an identity; then we apply to the iden-

* For the above particular case of Poincaré, an analogous proposition has been advanced by Chetaev (7) (without proof).

tity (10) the operation β_i and show that the obtained identity coincides with the equation, which we obtain by differentiating the second of the equations (11). Hence it follows that the quantities q_i and p_i defined by the equations (11) satisfy the canonical equations (8).

Here we make use of the following important property of Hamel's coefficients $\gamma_{\lambda, s, r}$, namely: Poisson's symbol of each pair of operators β_i is a linear form in all these operators, the coefficients of which are just Hamel's coefficients, i. e.

$$(\beta_i, \beta_\mu) = \beta_i(\beta_\mu) - \beta_\mu(\beta_i) = \sum_{\nu=1}^{k} \gamma_{i\nu\mu} \beta_\nu. \tag{12}$$

In the case when all $\gamma_{i\nu\mu} = \text{const}$, this property is known from the theory of Lie groups.

When, however, the $\gamma_{i\nu\mu}$ do depend on the q_i, the equation (12) may be easily deduced by using Schur's equation (6):

$$\sum_{\lambda=1}^{k} \left(\beta_{\rho\lambda} \frac{\partial \beta_{\sigma\nu}}{\partial q_\lambda} - \beta_{\sigma\lambda} \frac{\partial \beta_{\rho\nu}}{\partial q_\lambda} \right) = \sum_{\tau=1}^{n} \gamma_{\rho\tau\sigma} \beta_{\tau\nu}, \tag{13}$$

or by direct verification, using the expression of Hamel's coefficients

$$\gamma_{\lambda sr} = \sum_{\mu=1}^{k} \sum_{e=1}^{k} \beta_{\lambda\mu} \beta_{re} \left(\frac{\partial a_{s\mu}}{\partial q_e} - \frac{\partial a_{se}}{\partial q_\mu} \right)$$

and the conditions

$$\sum_{\mu=1}^{k} \alpha_{\rho\mu} \beta_{\lambda\mu} = \left\{ \begin{matrix} 1 & \text{for} & \rho = \lambda \\ 0 & \text{for} & \rho \neq \lambda \end{matrix} \right\}.$$

§ 4. In so far as the Boltzmann-Hamel equations written in the form (5) hold also for systems with non-holonomic linear constraints, the proposed generalization of the Hamilton-Jacobi theorem must be applicable also to non-holonomic systems.

In this case we must take from the equations (11) all equations of the first group, i. e. $\frac{\partial v}{\partial a_i} = b_i$, and those of the second group, which correspond to indices of holonomic co-ordinates, i. e.

$$p_\nu = \beta_\nu(v) \qquad (\nu = 1, 2, \ldots e).$$

At the same time for the determination of all $2k$ arbitrary constants a_i, b_i $(i = 1, 2, \ldots, k)$ we add g non-holonomic conditions substituting into them the derivatives of the q_i obtained from the equations $\frac{\partial v}{\partial a_i} = b_i$ and observing, of course, the initial conditions. Thus, adding the constraint equations, we obtain a system of equations for the determination of all co-ordinates q_i $(i = 1, 2, \ldots, k)$ and all a_i, b_i.

The calculations carried out for the simplest non-holonomic system — the sharp edged cutting wheel with two degrees of freedom, whose plane remains always orthogonal to the plane of motion (example of S. A. Tchaplyghin)—confirm the obtained results.

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PHYSICS

ON THE PROBLEM OF MASS OF AN ELEMENTARY PARTICLE

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Analysis of interactions leading to the formation of the proper mass of an elementary particle is one of the most serious problems of atomic physics. The statement of this problem, hitherto unsolved, has undergone considerable modification in the development of atomic physics. In the beginning of the XX century it was supposed that the problem in hand could be solved by Lorentz' electron theory. In accordance with this view, it was believed that every mass had an electromagnetic origin. Early attempts at solving the problem were made on the basis of conception of the electron as a volume electric charge, whose mass m_0 was determined by its charge e and its radius r_0. If c be velocity of light, then, according to order of magnitudes, we should be able to write

$$r_0 \sim e^2/mc^2.$$

Yet, further development of the theory has shown the volume electron to be incompatible with the requirements of relativity. At the same time the conceptions of point charge do not solve the problem, since they give rise to an infinitely great energy of the electron's proper field (1).

A consecutive relativistic invariant solution of the problem in terms of pre-quantum theory was outlined by G. Mie (2) and H. Weyl (3) who established the possibility of non-linear generalization of Lorentz electrodynamics which gives the proper mass of particles as a self-reaction of the field. In 1934 M. Born and L. Infeld (4), independently of Mie's theory, found Lagrange's function, permitting to obtain classically the proper mass of the electron. The pre-requisite, besides requirements of a relativist character, consisted also in the electromagnetic fields being finite, i. e. that fields of infinitely large tension could not be physically realized. However, these conceptions are of purely classical nature and their connexion with quantum ideas has so far not been sufficiently elucidated.

Passing over to the quantum theory, let us in first place note that non-relativistic quantum mechanics have been relativisated through Dirac's well-known equations, descriptive of a quantum particle possessing a charge and being under the influence of the outer field. The recourse made to this mechanical equation for describing an electro-dynamic process (which the electron virtually represents) has shown that the quantum conception involves, in the given case, a peculiar characteristic of the mass of a particle, rather difficult to understand from the viewpoint of the correspondence principle.

Namely, to speak roughly, we obtain here both the normal positive proper mass $+m_0c^2$ and the anomalous negative mass $-m_0c^2$, as it were two different types of particles are here described. It is known that it is just this peculiarity of the relativistic conception of mass which was made use of by Dirac [5] for the theory of the positron and the electron as the two possible states of a quantum particle. And here important rôle was taken by the fact these particles have anti-symmetrical statistics.

From the attempts to disentangle the problem of mass, we shall, besides Dirac's theory, also mention the following. H. Weyl [6], believing that mass is but a gravitational effect, suggested that the member with mass be altogether crossed out from Dirac's equation, so that it might be obtained by passing to the equation of the general relativity theory. This possibility was investigated by Schrödinger [7], Iwanenko [8] and Fock [9]. In result of these investigations it was found that the equation of second order for the general relativity theory had the following appearance (in terms of the usual notations):

$$\frac{1}{\sqrt{g}}\nabla_k\sqrt{g}g^{kl}\nabla_k\psi-\frac{R}{4}\psi-\frac{1}{2}f_{kl}s^{kl}\psi-\mu^2\psi=0,$$

where R is invariant of curvature, f_{kl} — outer field, $s^{kl}=\gamma^k\gamma^l-\gamma^l\gamma^k$, if γ^k are Dirac's matrices, $\mu=mc/\hbar$ is the mass member of Dirac's equation. This means there is truly a gravitational effect, but it is negligibly small in comparison with μ^2, as μ is of the order of 10^{11} cm^{-1}. Thus, the problem of the nature of μ remained unsolved.

In 1935 the author noted the unexpected connexion of this member with Lagrange's function of non-linear fields [10].

At the present time it is believed, first, that the mass of a quantum particle is not intrinsically bound up with an electromagnetic field, wherefore this is not an electrodynamic problem in the classical sense of the word, and secondly, that the problem of mass is connected with the introduction of constant r_0 into the quantum theory, its introduction involving a drastic change in quantum conceptions or, at least, a substantial limitation of their application. Besides, on the basis of certain sufficiently general considerations it is possible to foresee that this limitation of quantum conceptions involves a transition from linear equations, based on the principle of superposition and considered as equations of the classical theory of the «mechanical wave» field, to their non-linear modification. Thus, the mass of particles should be bound up not with its electromagnetic field, but with a reaction of its mechanical quantum field upon itself.

Thus, the purpose of this paper consists in showing that in Dirac's equation there is virtually contained the possibility of understanding mass in this way.

Let us recall that Dirac's equations [11] are obtained by a variation of wave-functions ψ_ρ, ψ_ρ^+ (a spinor of half-rank) from the variation principle

$$\delta\int L d\omega=0, \tag{1}$$

where

$$L=T^{\sigma}_{.\sigma}-\frac{m_0c}{\hbar}\bar{\psi}\alpha_0\psi. \tag{2}$$

Here $T^{\sigma}_{.\sigma}$ are components of the tensor energy matter, while

$$\psi^+\psi=\Omega=\sum_{\rho=1}^{4}\psi_\rho^+\psi_\rho \tag{3}$$

Ω is a relativistic invariant giving, with variation after ψ_ρ^+ or after ψ_ρ, a member with a mass in Dirac's equation for ψ_ρ or ψ_ρ^+. Leaving aside T_c^c, let us consider the second member in (1). Investigating its meaning L. De-Broglie ([12]) has shown it to be an invariant integral for the action of a particle passing into the classical integral, namely

$$-m_0 c\int^t dt\int\Omega\, dx\, dy\, dz \to -\int^t m_0 c\sqrt{1-\beta^2}\, dt,\quad \beta=\frac{v}{c} \tag{4}$$

where v is the velocity of particle. In order to make sure of (4), it is only necessary to substitute into (3) the magnitudes of ψ_ρ and ψ_ρ^+ for the curve of a monochromatic flat wave.

On the other hand, in 1935 the author ([13]) succeeded in showing that the same integral can be represented as an integral of the action of non-linear electrodynamics of the Born type. Let us now consider the connexion (2) with Lagrange's functions of field obtained in their time by Mie and Weyl and let us establish some consequences for a mass problem.

As is well known, in Mie's theory the field is described by two asymmetrical tensors (and tensor densities) H_{ik} and Φ_{ik}, then by two relativistic vectors s_k and φ^k, so that between Φ_{ik} and φ_k there exists a relation which is given for variation. At the same time H_{ik} and s^k are determined by variation as the functions of φ^k and Φ_{ik}, namely, as derivates of L after φ^k and Φ_{ik}:

$$\delta L=\frac{\partial L}{\partial g^{ik}}\delta g^{ik}+\frac{1}{2}H^{ik}\delta\Phi_{ik}-s^i\delta\varphi_i, \tag{5}$$

so that

$$H^{ik}=2\frac{\partial L}{\partial\varphi_{ik}}\quad\text{and}\quad s^i=-\frac{\partial L}{\partial\varphi_i}. \tag{6}$$

The equations for field are determined by the variation

$$\delta\int L d\omega=0. \tag{7}$$

Further, between the tensors of energy matter and the fields there exists the relationship

$$T_i^k=H^{kr}\Phi_{ir}-s^k\varphi_i-\frac{1}{2}L\delta_i^k, \tag{8}$$

while, if gravitational field be absent, then

$$\frac{\partial T_i^k}{\partial x_k}=0. \tag{9}$$

Let us now show that the integral of action from (3) and (4) may be expressed similarly, i. e. without introducing any limitations for ψ_ρ and ψ_ρ^+, through two anti-symmetric tensors and two relativistic vectors, just as in the theory of the type expounded by Mie.

Let ψ_ρ and ψ_ρ^+ be certain spinors of half-rank (not necessarily the solutions of Dirac's equation). Then, making from them bilinear forms, we obtain quantities, following ordinary tensor laws of transformation. Of eight spinors ψ_ρ, ψ_ρ^+ it is possible, as we know, to form 16 quantities, to the number of which the invariant Ω also belongs; namely, we shall find in this way two invariants, one anti-symmetrical tensor, one relativistic vector and one anti-symmetrical tensor of the third rank. Between the quantities so obtained, taking, as is well known, a fundamental part in Dirac's equations, there exist identical relationships, namely, that the

number of independent values is equal to six. All these relations were obtained by the author ([15]) in 1930 and quite independently by Zaycoff ([14]) in 1931, and we shall now make use of them, resorting to the form which Zaycoff has imparted to these relations.

Let

$$\alpha_k,\ \alpha_k^+ \quad (k=1,2,3,0)$$

Dirac's matrix is such that

$$\left.\begin{aligned} &\alpha_m^+\alpha_r+\alpha_r^+\alpha_m=\alpha_m\alpha_r^++\alpha_r\alpha_m^+=2\delta_{mr} \\ \text{and}\qquad &\alpha=\pm i\alpha_k\alpha_l^+\alpha_m\alpha_n^+\alpha_0,\quad k\neq l\neq m\neq n \end{aligned}\right\} \tag{10}$$

are also an anti-commutative matrix. Then

$$\Omega_0=\bar{\psi}\alpha_0\psi \text{ and } \Omega_1=\bar{\psi}\alpha\psi-\text{invariants},$$
$$(k\neq l)\,\Phi_{kl}=i\bar{\psi}\alpha_k\alpha_l^+\alpha_0\psi-\text{anti-symmetrical tensor},$$
$$\varphi_k=\bar{\psi}\alpha_k\psi,\quad k=1,2,3,4-\text{relativistic vector}.$$

Further

$$\Phi_{kr}\Phi_{kr}-2\Omega_0^2+2\Omega_1^2\equiv 0, \tag{11}$$

$$\varphi_r\varphi_r+\Omega_0^2+\Omega_1^2\equiv 0 \quad \text{etc.} \tag{12}$$

Hence

$$\Omega_0\equiv\frac{1}{2}\sqrt{\Phi_{kr}\Phi_{kr}-2\varphi_r\varphi_r}, \tag{13}$$

Φ_{kr} being expressed through φ_r. (13) may be written in the form:

$$\Omega_0\equiv\frac{1}{2}H^{ik}\Phi_{ik}-s^i\varphi_i, \tag{14}$$

where

$$H^{ik}=\frac{\Phi_{ik}}{\sqrt{\Phi_{kr}\Phi_{kr}-2\varphi_r\varphi_r}},\quad s^i=-\frac{\varphi_i}{\sqrt{\Phi_{kr}\Phi_{kr}-2\varphi_r\varphi_r}} \tag{15}$$

while (cf. 6)

$$H^{ik}=2\frac{\partial\Omega}{\partial\Phi_{ik}} \quad \text{and} \quad s^i=-\frac{\partial\Omega}{\partial\varphi_i}. \tag{16}$$

We shall not consider here the very important connexion between Ω, L and tensor T_σ^ε [analogous to (8)], but shall only consider expression $\delta\Omega$ obtained with variation after ψ and ψ^+, i. e. let us consider the new form of the member determining mass in Dirac's equation.

By varying Ω_0 we obtain from (14)

$$\delta\Omega_0=\frac{1}{2}\delta H^{ik}\Phi_{ik}+\frac{1}{2}H^{ik}\delta\Phi_{ik}-\delta s^i\varphi_i-s^i\delta\varphi_i. \tag{17}$$

Using Φ_{kl} and φ_k and (15), we find:

$$\delta\Phi_{kl}=i\,(\delta\bar{\psi})\,\alpha_k\alpha_l^+\alpha_0\psi,\quad \delta\varphi_i=(\delta\bar{\psi})\,\alpha_i\psi \tag{18}$$

and

$$\delta H^{ik}=\delta\left(\frac{\Phi_{ik}}{\sqrt{\dots}}\right)=\frac{\delta\Phi_{ik}\sqrt{\dots}-\Phi_{ik}\delta\sqrt{\dots}}{(\sqrt{\dots})^2}. \tag{19}$$

But

$$\delta\sqrt{\dots}=(\delta\bar{\psi})\,\alpha_0\psi \tag{20}$$

and

$$\delta H^{ik}\Phi_{ik}=H^{ik}i\,(\delta\bar{\psi})\,\alpha_i\alpha_k^+\alpha_0\psi-H^{ik}H_{ik}\,(\delta\bar{\psi})\,\alpha_0\psi. \tag{21}$$

Further

$$\delta s^i = \delta\left(-\frac{\psi_i}{\sqrt{\ldots}}\right) = -\frac{\delta\psi_i(\sqrt{\ldots}) - \varphi_i\delta(\sqrt{\ldots})}{(\sqrt{\ldots})^2}, \tag{22}$$

$$-\varphi_i\delta s^i = +(\delta\bar{\psi})\,\alpha_i\psi s^i - s^i s_i(\delta\bar{\psi})\,\alpha_0\psi. \tag{23}$$

Thus,

$$\delta\Omega_0 = \left\{\frac{1}{2}H^{ik}i\alpha_i\alpha_k^+\alpha_0\psi - \frac{1}{2}H^{ik}H_{ik}\alpha_0\psi + \right.$$

$$\left. + \frac{1}{2}H^{ik}i\alpha_i\alpha_k^+\alpha_0\psi + s^i\alpha_i\psi - s^i s_i\alpha_0\psi - s^i\alpha_i\psi\right\}\delta\bar{\psi}. \tag{24}$$

In result we obtain the following expression for the last term in the equation of the electron:

$$\frac{mc}{\hbar}\left(H^{ik}i\alpha_i\alpha_k^+\alpha_0\psi - \left(\frac{1}{2}H^{ik}H_{ik} + s^i s_i\right)\alpha_0\psi\right). \tag{25}$$

We see that this member describes the reaction of the proper field of mechanical waves upon a particle, consisting of two terms of virtually different nature. The first is very analogous to the spin-member available in the equation of the second order, with the difference that instead of an outer electromagnetic field there stands in it a field of material waves H^{ik}. The second term gives, as it were, a Lagrangian for the field. Both are non-linear in ψ, but compensating each other they give a linear member.

Let us note in conclusion that, while using this method for the transformation of the mass member in Dirac's equation, it is possible to modify it reasonably, say, by crossing out these or other members, so as to transform it into the sought non-linear equation for mechanical fields.

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PHYSICS

ON THE THEORY OF THE RELATIVISTIC QUANTUM PARTICLE

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1. Dirac's well-known equation determining the state of a particle under the effect of an electromagnetic field is at present the only basis for the theory of relativistic quantum particles having an electrical charge. These equations, in their original form, represent mechanical quantum equations of a relativistic quantum p r o b l e m o f a s i n g l e b o d y. Relativistic quantum problems can by now be definitely regarded on principal grounds as problems for an indefinite number of particles. It is therefore necessary to formulate Dirac's equation in such a way as to make possible a natural transition from a problem of a single body to a problem with an indefinite (in a quantum sense) number of particles. This transition is, as is well known, partly being accomplished by the so-called «method of secondary quantization» and implies a consideration of the interaction of particles by means of an electromagnetic field also regarded as a quantum system. Without referring for the present to the method of secondary quantization, we shall note the fact that Dirac's equations, being quantum equations of motion, describe the m e c h a n i c a l properties of an electron, but at the same time they also express the properties of the electron as an e l e c t r o d y n a m i c s y s t e m. This aspect of Dirac's equations is expressed, for instance, by an altogether peculiar «splitting» of dynamic magnitudes into «mechanic» and «electrodynamic» ones. Thus, for velocity we have operator dx_k/dt and operator with α_k; we have operators for the magnetic moment of electron, etc. Thus, Dirac's equations may not only be considered as mechanical equations of motion, but also as e q u a t i o n s o f f i e l d. This point of view is expedient in dealing with questions of the further development of the quantum theory with a view to its further extension to problems connected with the mass of particles.

The peculiarity of Dirac's equations consists in the fact that they are not expressed by means of the usual tensor apparatus of the relativity theory, but by means of spinor relations (2). It is, however, possible and expedient to find a formulation of the theory in ordinary tensor relations in order to compare Dirac's equation with those of other theories of the field. For this purpose it will be necessary to pass from ψ_ρ and ψ_ρ^* and their derivatives to tensors and invariants formed of them and to examine the relations existing between them on account of Dirac's equations. The system of tensor equations obtained in this way can be considered on the general principles used for equations of field theories and may then be subjected to quantization

and non-linear modification for the purpose of estimating the problem of mass.

The problem set in such a form will be solved, provided there will be found Lagrange's function corresponding to the system of equations* obtained in this way and depending upon such quantities, as would describe by means of a system the electron as an electrodynamic system. Moreover, the invariability of Lagrange's integral with reference to the relativistic transformations of co-ordinates and time always affords the possibility to find the energy tensor of the matter of this system and jointly with it also the laws of the conservation of energy and impulse, as well as other conservation laws.

In this paper we shall but partly examine the first part of the problem: namely, we are to find certain tensors and invariants and the equations connecting them, which are characteristic of Dirac's electron.

2. Resorting to the following direct though somewhat cumbersome method, we obtain the tensor equations sought. Let us take Dirac's equation in the commonly known form (3):

$$\left[\sum_{\mu=1}^{4}\gamma_\mu\left(\frac{\partial}{\partial x_\mu}+\frac{ie}{\hbar c}\varphi_\mu\right)+\frac{mc}{\hbar}\right]\psi=0, \qquad \text{(A)}$$

$$\left[\sum_{\mu=1}^{4}\tilde{\gamma}_\mu\left(\frac{\partial}{\partial x_\mu}-\frac{ie}{\hbar c}\varphi_\mu\right)-\frac{mc}{\hbar}\right]\psi^+=0, \qquad \text{(B)}$$

where $\sim$ denotes the transposed matrix. Multiplying the first equation by $\psi^+\alpha$, and the second by $\psi\tilde{\alpha}$ with α of one of the 16 matrices

$$1,\quad \gamma_k,\quad \gamma_k\gamma_l,\quad \gamma_5\gamma_k\gamma_l,\quad \gamma_5 \qquad k;\ l=(1,2,3,4)$$

and adding up and subtracting the obtained equations, we shall find 32 equations, whose analysis enables us to impart the tensor form to Dirac's theory. This is accounted for by the fact that ψ and ψ^+ are spinors of the half rank and accordingly quantities that are bilinear in ψ, ψ^+ and their derivatives have tensor properties. Let us calculate consecutively all 32 equations.

Multiplying (A) by ψ^+ and (B) by ψ and adding up, we obtain the known equation for current of charges

$$\sum_{\mu=1}^{4}\frac{\partial}{\partial x_\mu}(\psi^+\gamma_\mu\psi)=0. \qquad (1)$$

But by subtracting, we have the equation which determines invariant $\psi^+\psi$:

$$\sum_{\mu=1}^{4}\left(\psi^+\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^+}{\partial x_\mu}\gamma_\mu\psi+\frac{2ie}{\hbar c}\varphi_\mu\psi^+\gamma_\mu\psi\right)+\frac{2mc}{\hbar}\psi^+\psi=0. \qquad (2)$$

* Let us, moreover, remark that this system of equations represents a solution of the problem of describing the quantum system by means of «Eichinvariant» quantities, i. e. experimentally measurable quantities. Such quantity as ψ_ρ, as well as potentials, are obviously not comprised in this system of equations, being included in «Eichinvariant» quantities and determined by them.

Multiplying (A) by $\psi^+\gamma_k$ and (B) by $\tilde{\psi}\tilde{\gamma}_k$ and then adding up, we obtain (no adding up according to Latin indices is effected):

$$\sum_{\mu=1}^{4}\left[\psi^+\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}+\tilde{\psi}\tilde{\gamma}_k\tilde{\gamma}_\mu\frac{\partial\psi^+}{\partial x_\mu}+\frac{ie}{\hbar c}\varphi_\mu[\psi^+\gamma_k\gamma_\mu\psi-\tilde{\psi}\tilde{\gamma}_k\tilde{\gamma}_\mu\psi^+]\right]+$$
$$+\frac{mc}{\hbar}(\psi^+\gamma_k\psi-\tilde{\psi}\tilde{\gamma}_k\psi^+)=0,$$

since

$$a\tilde{\gamma}b=b\gamma a \quad \text{and} \quad \tilde{\gamma}_\alpha\tilde{\gamma}_\beta=\tilde{\gamma}_\beta\tilde{\gamma}_\alpha,$$

where γ is any matrix; then the equation will be rewritten as follows:

$$\sum_{\mu=1}^{4}\left[\psi^+\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}+\frac{\partial\psi^+}{\partial x_\mu}\gamma_\mu\gamma_k\psi+\frac{ie}{\hbar c}\varphi_\mu[\psi^+\gamma_k\gamma_\mu\psi-\psi^+\gamma_\mu\gamma_k\psi]\right]+$$
$$+\frac{mc}{\hbar}(\psi^+\gamma_k\psi-\psi^+\gamma_k\psi)=0.$$

Making use of the anti-commutativeness of $\gamma_k\ \gamma_k\gamma_l+\gamma_l\gamma_k=2\delta_{kl}$, we find four equations:

$$\sum_{\mu\neq k}^{4}\left(\psi^+\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^+}{\partial x_\mu}\gamma_k\gamma_\mu\psi+\frac{2ie}{\hbar c}\varphi_\mu\psi^+\gamma_k\gamma_\mu\psi\right)+$$
$$+\frac{\partial\psi^+\psi}{\partial x_k}=0 \quad (k=1,2,3,4). \qquad (3-6)$$

The operation of subtraction gives in the present case

$$\sum_{\mu=1}^{4}\left[\psi^+\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^+}{\partial x_\mu}\gamma_\mu\gamma_k\psi+\frac{ie}{\hbar c}\varphi_\mu[\psi^+\gamma_k\gamma_\mu\psi+\psi^+\gamma_\mu\gamma_k\psi]\right]+$$
$$+\frac{2mc}{\hbar}\psi^+\gamma_k\psi=0,$$

i. e. four equations more:

$$\sum_{\mu\neq k}^{4}\left[\frac{\partial\psi^+\gamma_k\gamma_\mu\psi}{\partial x_\mu}\right]+\left(\psi^+\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^+}{\partial x_k}\psi+\frac{2ie}{\hbar c}\varphi_k\psi^+\psi\right)+$$
$$+\frac{2mc}{\hbar}\psi^+\gamma_k\psi=0. \qquad (7-10)$$

Multiplying (A) by $\psi^+\gamma_k\gamma_l$ and (B) by $\tilde{\psi}\tilde{\gamma}_l\tilde{\gamma}_k$, subtracting and adding up, we obtain

$$\sum_{\mu=1}^{4}\left[\psi^+\gamma_k\gamma_l\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\mp\tilde{\psi}\tilde{\gamma}_l\tilde{\gamma}_k\tilde{\gamma}_\mu\frac{\partial\psi^+}{\partial x_\mu}+\frac{ie}{\hbar c}\varphi_\mu(\psi^+\gamma_k\gamma_l\gamma_\mu\psi\pm\tilde{\psi}\tilde{\gamma}_l\tilde{\gamma}_k\tilde{\gamma}_\mu\psi^+)\right]+$$
$$+\frac{mc}{\hbar}[\psi^+\gamma_k\gamma_l\psi\pm\tilde{\psi}\tilde{\gamma}_k\tilde{\gamma}_l\psi^+]=0;$$

since

$$a\tilde{\gamma}_l\tilde{\gamma}_k\tilde{\gamma}_\mu b=a(\tilde{\gamma}_\mu\tilde{\gamma}_k\tilde{\gamma}_l)b=b(\gamma_\mu\gamma_k\gamma_l)a,$$

we obtain

$$\sum_{\mu=1}^{4}\left[\psi^+\gamma_k\gamma_l\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\pm\frac{\partial\psi^+}{\partial x_\mu}\gamma_\mu\gamma_k\gamma_l\psi+\frac{ie}{\hbar c}\varphi_\mu(\psi^+\gamma_k\gamma_l\gamma_\mu\psi\mp\psi^+\gamma_\mu\gamma_k\gamma_l\psi)\right]+$$
$$+\frac{mc}{\hbar}[\psi^+\gamma_k\gamma_l\psi\pm\psi^+\gamma_l\gamma_k\psi]=0.$$

Taking the upper sign, we obtain the following equations:

$$\sum_{\mu\neq k,\, l}^{4}\left[\frac{\partial\psi^{+}\gamma_k\gamma_l\gamma_\mu\psi}{\partial x_\mu}\right]+\left(\psi^{+}\gamma_k\frac{\partial\psi}{\partial x_l}-\frac{\partial\psi^{+}}{\partial x_l}\gamma_k\psi+\frac{2ie}{\hbar c}\varphi_l\psi^{+}\gamma_k\psi\right)-$$
$$-\left(\psi^{+}\gamma_l\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^{+}}{\partial x_k}\gamma_l\psi+\frac{2ie}{\hbar c}\varphi_k\psi^{+}\gamma_l\psi\right)=0, \qquad (11-16)$$

$k, l=1, 2, 3, 4$; $(k, l)=(1, 4), (2, 4), (3, 4), (1, 3), (2, 3), (1, 2)$.

Taking the lower sign, we obtain:

$$\sum_{\mu\neq k,\, l}^{4}\left[\psi^{+}\gamma_k\gamma_l\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_k\gamma_l\gamma_\mu\psi+\frac{2ie}{\hbar c}\varphi_\mu\psi^{+}\gamma_k\gamma_l\gamma_\mu\psi\right]+$$
$$+\frac{\partial\psi^{+}\gamma_k\psi}{\partial x_l}-\frac{\partial\psi^{+}\gamma_l\psi}{\partial x_k}+\frac{2mc}{\hbar}\psi^{+}\gamma_k\gamma_l\psi=0. \qquad (17-22)$$

Multiplying by $\psi^{+}\gamma_5\gamma_k$ and $\psi\tilde{\gamma}_k\tilde{\gamma}_5$ respectively and then adding up and subtracting, we obtain:

$$\sum_{\mu=1}^{4}\left[\psi^{+}\gamma_5\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\mp\psi\tilde{\gamma}_k\tilde{\gamma}_5\tilde{\gamma}_\mu\frac{\partial\psi^{+}}{\partial x_\mu}+\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi\frac{ie}{\hbar c}\varphi_\mu\pm\right.$$
$$\left.\pm\psi\tilde{\gamma}_k\tilde{\gamma}_5\tilde{\gamma}_\mu\psi^{+}\frac{ie}{\hbar c}\varphi_\mu\right]+\frac{mc}{\hbar}(\psi^{+}\gamma_5\gamma_k\psi^{+}\pm\psi\tilde{\gamma}_k\tilde{\gamma}_5\psi)=0;$$

$$\sum_{\mu=1}^{4}\left[\psi^{+}\gamma_5\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\mp\frac{\partial\psi^{+}}{\partial x}\gamma_\mu\gamma_5\gamma_k\psi+\frac{ie}{\hbar c}\varphi_\mu\psi^{+}\left[(\gamma_5\gamma_k\gamma_\mu\psi)\pm\psi^{+}\gamma_\mu\gamma_5\gamma_k\psi\right]\right]+$$
$$+\frac{mc}{\hbar}(\psi^{+}\gamma_5\gamma_k\psi\pm\psi^{+}\gamma_5\gamma_k\psi)=0.$$

Segregating the term with $k=\mu$, we obtain:

$$\psi^{+}\gamma_5\frac{\partial\psi^{+}}{\partial x_k}\pm\frac{\partial\psi^{+}}{\partial x_k}\gamma_5\psi+\frac{ie}{\hbar c}\varphi_k(\psi^{+}\gamma_5\psi\mp\psi^{+}\gamma_5\psi)+$$
$$+\sum_{\mu\neq k}^{4}\left[\psi^{+}\gamma_5\gamma_k\gamma_\mu\frac{\partial\psi^{+}}{\partial x_\mu}\mp\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_5\gamma_k\gamma_\mu\psi+\frac{ie}{\hbar c}\varphi_\mu(\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi\pm\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi)\right]+$$
$$+\frac{mc}{\hbar}(\psi^{+}\gamma_5\gamma_k\psi\pm\psi^{+}\gamma_5\gamma_k\psi)=0.$$

Taking the upper sign, we find:

$$\frac{\partial\psi^{+}\gamma_5\psi}{\partial x_k}+\sum_{\mu\neq k}^{4}\left(\psi^{+}\gamma_5\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_5\gamma_k\gamma_\mu\psi+\frac{2ie}{\hbar c}\varphi_\mu\psi^{+}\gamma_5\gamma_k\psi\right)+$$
$$+\frac{2mc}{\hbar}\psi^{+}\gamma_5\gamma_k\psi=0, \qquad (k=1, 2, 3, 4). \qquad (23-26)$$

Taking the lower sign:

$$\sum_{\mu\neq k}^{4}\frac{\partial\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi}{\partial x_\mu}+\left(\psi^{+}\gamma_5\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^{+}}{\partial x_k}\gamma_5\psi+\frac{2ie}{\hbar c}\varphi_k\psi^{+}\gamma_5\psi\right)=0, \qquad (27-30)$$
$$k=(1, 2, 3, 4).$$

Finally, multiplying (A) and (B) by $\psi^{+}\gamma_5$ and $\psi\tilde{\gamma}_5$ respectively, adding or subtracting, we obtain:

$$\sum_{\mu=1}^{4}\left[\psi^{+}\gamma_5\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\pm\psi\tilde{\gamma}_5\tilde{\gamma}_\mu\frac{\partial\psi^{+}}{\partial x_\mu}+\frac{ie}{\hbar c}\varphi_\mu(\psi^{+}\gamma_5\gamma_\mu\psi\mp\psi\tilde{\gamma}_5\tilde{\gamma}_\mu\psi^{+})\right]+$$
$$+\frac{mc}{\hbar}(\psi^{+}\gamma_5\psi\mp\psi\tilde{\gamma}_5\psi^{+})=0,$$

i. e. the following two equations:

$$\sum_{\mu=1}^{4}\Big[\psi^{+}\gamma_5\gamma_\mu\frac{\partial\psi}{\partial x_\mu}\mp\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_5\gamma_\mu\psi+\frac{ie}{\hbar c}\varphi_\mu(\psi^{+}\gamma_5\gamma_\mu\psi\pm\psi^{+}\gamma_5\gamma_\mu\psi)\Big]+$$

$$+\frac{mc}{\hbar}(\psi^{+}\gamma_5\psi\mp\psi^{+}\gamma_5\psi)=0,$$

i. e.

$$\sum_{\mu=1}^{4}\Big(\psi^{+}\gamma_5\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_5\gamma_\mu\psi+\frac{2ie}{\hbar c}\varphi_\mu\psi^{+}\gamma_5\gamma_\mu\psi\Big)=0. \tag{31}$$

$$\sum_{\mu-1}^{4}\Big(\frac{\partial\psi^{+}\gamma_5\gamma_\mu\psi}{\partial x_\mu}\Big)+\frac{2mc}{\hbar}\psi^{+}\gamma_5\psi=0. \tag{32}$$

3. Let us examine the 32 equations obtained. Part of them was found by different authors in a more or less formal way. Thus, equation (1) was deduced by Dirac [4] and is known in electron theory as the law of the conservation of electric charges. Equation (32) was obtained by Uhlenbeck and Laporte [5] together with certain other equations. Further, some of the equations were suggested by F. Wiesniewsky [6] who considered Dirac's equation in connexion with Maxwell's equation. Finally, relations (11 — 16) have been made use of by Tetrode [7], Weyl [8], Rosenfeld [9] and others in considering the problem of the tensor of the energy and matter, remaining however so far not fully settled. Referring to the equations, we obtain in the first place four relativistic vectors, three of which satisfy the equations of conservation, while the fourth satisfies the equation of «non-conservation» established by Uhlenbeck and Laporte, namely:

$$\sum_{\mu=1}^{4}\frac{\partial}{\partial x_\mu}(\psi^{+}\gamma_\mu\psi)=0, \qquad (1) \quad (\alpha_1)$$

$$\sum_{k=1}^{4}\frac{\partial}{\partial x_k}\Big(\psi^{+}\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^{+}}{\partial x_k}\psi+\frac{2ie}{\hbar c}\varphi_k\psi^{+}\psi\Big)=0, \qquad (\alpha_2)$$

$$\sum_{k=1}^{4}\frac{\partial}{\partial x_k}\Big(\psi^{+}\gamma_5\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^{+}}{\partial x_k}\gamma_5\psi+\frac{2ie}{\hbar c}\varphi_k\psi^{+}\gamma_5\psi\Big)=0, \qquad (\alpha_3)$$

$$\sum_{k=1}^{4}\frac{\partial}{\partial x_k}(\psi^{+}\gamma_5\gamma_\mu\psi)+\frac{2mc}{\hbar}\psi^{+}\gamma_5\psi=0. \qquad (32) \quad (\alpha_4)$$

Let us denote these relativistic vectors by U_k, I_k, $I_{5,k}$, $U_{5,k}$ respectively. U_k, I_k, $I_{5,k}$, U_{5k} are connected with antisymmetric tensors, and accordingly the following equations will be available:

$$-\frac{2mc}{\hbar}U_k=I_k+\sum_{\mu\neq k}^{4}\frac{\partial\psi^{+}\gamma_k\gamma_\mu\psi}{\partial x_\mu}, \quad k=1,2,3,4. \qquad (\beta_1)$$
(equations 7 — 10)

$$I_{5,k}=-\sum_{\mu\neq k}^{4}\frac{\partial\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi}{\partial x_\mu}, \quad k=1,2,3,4. \qquad (\beta_2)$$
(equations 27 — 30)

and

$$-\frac{2mc}{\hbar}U_{5,k}=\sum_{\mu\neq k}^{4}\left(\psi^{+}\gamma_5\gamma_k\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_5\gamma_k\gamma_\mu\psi+\right.$$

$$\left.+\frac{2ie}{\hbar c}\varphi_\mu\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi\right)+\frac{\partial\psi^{+}\gamma_5\psi}{\partial x_k}. \qquad (\beta_3)\ \text{(equations 23—26)}$$

Two anti-symmetric tensors $\psi^{+}\gamma_k\gamma_\mu\psi$ and $\psi^{+}\gamma_5\gamma_\mu\psi$ connected with I_k and $I_{5,k}$ with respect to equations (7—10) and (27—30) are d u a l tensors on account of the relation

$$\gamma_1\gamma_2\gamma_3\gamma_4=\alpha\gamma_5, \qquad (33)$$

where α is a number.

The tables of the components of these tensors are connected with one another in the following manner (we write out only one half of the components because of their anti-symmetry):

$$\begin{bmatrix} 0; & \gamma_5\gamma_1\gamma_2; & \gamma_5\gamma_1\gamma_3; & \gamma_5\gamma_1\gamma_4 \\ & 0; & \gamma_5\gamma_2\gamma_3; & \gamma_5\gamma_2\gamma_4 \\ & & 0; & \gamma_5\gamma_3\gamma_4 \\ & & & 0 \end{bmatrix}=\begin{bmatrix} 0; & -\alpha\gamma_3\gamma_4; & +\alpha\gamma_2\gamma_4; & -\alpha\gamma_2\gamma_3 \\ & 0; & -\alpha\gamma_1\gamma_4; & +\alpha\gamma_1\gamma_3 \\ & & 0; & -\alpha\gamma_1\gamma_2 \\ & & & 0 \end{bmatrix} \qquad (34)$$

Let us recall that the dual tensors Φ_{kl} and Φ^{*}_{kl} are interconnected in such a way that the divergence of the one serves as a «cycle» of the other, namely:

$$\frac{\partial\Phi_{ik}}{\partial x^l}+\frac{\partial\Phi_{li}}{\partial x^k}+\frac{\partial\Phi_{kl}}{\partial x^i}=\frac{\partial\Phi^{*}_{ik}}{\partial x^k}. \qquad (35)$$

Thus, we can replace in (β_1) the divergence of the tensor

$$\Phi_{k\mu}=\psi^{+}\gamma_k\gamma_\mu\psi \qquad (\gamma_1)$$

by a «cycle» (four-dimensional rotor) of the dual tensor

$$\Phi^{*}_{k\mu}=\Phi_{5k\mu}=\psi^{+}\gamma_5\gamma_k\gamma_\mu\psi. \qquad (\gamma_2)$$

It is also possible to combine (β_1) and (β_2) into a single system of equations of the Maxwell type with one of the tensors (γ_1) or (γ_2) [cf. Wiesniewsky (6)].

Further, we obtain two anti-symmetrical tensors, namely:

$$T_{kl}=\left(\psi^{+}\gamma_k\frac{\partial\psi}{\partial x_l}-\frac{\partial\psi^{+}}{\partial x_l}\gamma_k\psi+\frac{2ie}{\hbar c}\varphi_l\psi^{+}\gamma_k\psi\right)-$$

$$-\left(\psi^{+}\gamma_l\frac{\partial\psi}{\partial x_k}-\frac{\partial\psi^{+}}{\partial x_k}\gamma_l\psi+\frac{2ie}{\hbar c}\varphi_k\psi^{+}\gamma_l\psi\right)=$$

$$=-\sum_{\mu\neq k,\,l}\frac{\partial\psi^{+}\gamma_k\gamma_l\gamma_\mu\psi}{\partial x_\mu} \qquad (\gamma_3)\ \text{(equations 11—16)}$$

and

$$\Theta_{kl}=\frac{\partial\psi^{+}\gamma_k\psi}{\partial x_l}-\frac{\partial\psi^{+}\gamma_l\psi}{\partial x_k}=-\sum_{\mu\neq k,\,l}^{4}\left(\psi^{+}\gamma_k\gamma_l\gamma_\mu\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi^{+}}{\partial x_\mu}\gamma_\mu\gamma_l\gamma_k\psi+\right.$$

$$\left.+\frac{2ie}{\hbar c}\varphi_\mu\psi^{+}\gamma_k\gamma_l\gamma_\mu\psi\right)-\frac{2mc}{\hbar}\psi^{+}\gamma_k\gamma_l\psi, \qquad (\gamma_4)\ \text{(equations 17—22)}$$

having

$$\sum_{l=1}^{4}\frac{\partial T_{kl}}{\partial x_l}=0, \quad \text{i. e.} \quad \frac{\partial T^{*}_{ik}}{\partial x^l}+\frac{\partial T^{*}_{li}}{\partial x^k}+\frac{\partial T^{*}_{kl}}{\partial x^i}=0, \qquad (\delta_1)$$

and

$$\frac{\partial \Theta_{ik}}{\partial x^l}+\frac{\partial \Theta_{li}}{\partial x^k}+\frac{\partial \Theta_{kl}}{\partial x^i}=0. \qquad (\delta_2)$$

The anti-symmetrical tensor T_{kl}, or the dual tensor T^*_{kl}, stand in direct relationship to the energy tensor of matter of Dirac's equation, as was pointed out by Tetrode.

Equation (γ_4) is remarkable by the fact of its connecting U_k with tensor Φ_{kl} which, in its turn, determines vector U_k by equations (β_λ). Assuming

$$\Theta_{kl}=S_{kl}-\frac{2mc}{\hbar}\Phi_{kl}, \qquad (\gamma_4')$$

and making use of (δ_2), we find:

$$\frac{\partial S_{kl}}{\partial x^l}+\frac{\partial S_{li}}{\partial x^k}+\frac{\partial S_{kl}}{\partial x^i}=-\frac{2mc}{\hbar}\left(\frac{\partial \Phi_{kl}}{\partial x^l}+\frac{\partial \Phi_{li}}{\partial x^k}+\frac{\partial \Phi_{kl}}{\partial x^i}\right).$$

But the right-hand side of this equation is equal, with a precision up to the constant multiplier $\frac{2mc}{\hbar}$, to the right-hand side of the equation (β_2), and consequently we obtain the following equation:

$$I_{5,\,k}=\frac{\hbar}{2mc}\left(\frac{\partial S_{kl}}{\partial x^l}+\frac{\partial S_{li}}{\partial x^k}+\frac{\partial S_{kl}}{\partial x^i}\right), \qquad (\beta_4')$$

connecting current $I_{5,\,k}$ with anti-symmetrical tensor S_{kl}. Further, tensor T_{kl} is expressed through vector $U_{5,\,k}$ satisfying Uhlenbeck-Laporte's equation (α_4).

In fact, on account of (33), vector $U_{k,\,5}$ can be written with the help of quantities $\psi^+\gamma_k\gamma_l\gamma_\mu\psi$ standing in the right-hand side of equation (γ_3), namely vector $U_{k,\,5}$ represents an anti-symmetrical tensor of the third rank with

$$\left.\begin{aligned}\gamma_1\gamma_2\gamma_3&=\alpha\gamma_5\gamma_4 \qquad -\gamma_2\gamma_3\gamma_4=\alpha\gamma_5\gamma_1\\ \gamma_1\gamma_3\gamma_4&=\alpha\gamma_5\gamma_2 \qquad -\gamma_1\gamma_2\gamma_4=\alpha\gamma_5\gamma_3\end{aligned}\right\} \qquad (36)$$

Thus, equation (γ_3) and (α_4) form a system determining $U_{5,\,k}$ by T_{kl} and the invariant $\psi^+\gamma_5\psi$. If we introduce the notations (H and E) for the components of tensor $\Phi_{k\mu}$ and if, next, we represent by a line drawn above the respective notation of quantity of the type

$$T_{\lambda\mu}=\text{const}\left(\bar{\psi}\alpha_\lambda\frac{\partial\psi}{\partial x_\mu}-\frac{\partial\psi}{\partial x_\mu}\alpha_\lambda\psi+2i\,\varphi_\mu\bar{\psi}\alpha_\lambda\psi_\varphi\right), \qquad (37)$$

we can give to the system of equations the following simple form:

$$\left.\begin{aligned}&-\overline{(\psi^+\psi)}_t+\operatorname{div}H=0,\\ &-\frac{\partial H}{\partial x_4}+\overline{\operatorname{grad}1}=\operatorname{curl}E,\\ &(\gamma_5)+\operatorname{div}E+\lambda U_4=0,\\ &\frac{\partial E}{\partial x_4}+\overline{\operatorname{grad}\gamma_5}+\lambda U=\operatorname{curl}H,\\ &\frac{\partial U_{5,k}}{\partial x_4}+\overline{\operatorname{grad}U_{5,\,k}}+\overline{\operatorname{curl}U}=0,\\ &\overline{\frac{\partial U_{5,k}}{\partial x_4}}+\overline{\operatorname{grad}U_{5,\,k}}+\operatorname{curl}U=\lambda H,\\ &-\frac{\partial x_4}{\partial x_4}+\operatorname{grad}U_4+\overline{\operatorname{curl}U_{5,\,k}}=\lambda E,\\ &-\overline{\frac{\partial u}{\partial t}}-\overline{\operatorname{grad}U_4}+\operatorname{curl}U_{5,\,k}=0 \text{ etc.}\end{aligned}\right\} \qquad (38)$$

These relations are in a great measure analogous to the relations of the theory of field Mie. To these relations we must now add equations comprising invariants $\psi^+\psi$ and $\psi^+\gamma_5\psi$, namely equations (2) determining Lagrange's function of Dirac's equations of the first order, etc.

We see in the first place that two types of magnitudes enter the equations: bilinear in ψ and ψ^+ and then those of Hermit's form of type (37) containing derivatives from ψ and ψ^+. All these quantities are «Eichinvariant» quantities, i. e. measurable quantities.

Let us, furthermore, note the following: among bilinear forms in ψ, ψ^+ there exist identities ([10]), altogether independent of the choice of ψ and ψ^+ and all of them liable to be expressed in terms of 6 independent forms (the total number of forms amounts to 16). There are, likewise, identical relations between quantities of the type of (37), these also being bilinear forms in ψ and $f_{\alpha\beta}=\frac{\partial\psi_\alpha}{\partial x_\beta}$. However, these relations have not yet been investigated. Thus, in order to give a solution of the problem, to the statement of which this paper is devoted (i. e. to the problem of formulating Dirac's equations in the form of tensor equations and their comparison, being understood as the equations of the theory of the field, with equations of other theories of the field), all identities between $T_{\lambda\mu}$ have to be found. Having expressed all of them through independent identities, we shall be able to find equations for the latter. It should however be noted that this problem may be solved by means of the method of homogeneous hexaspherical co-ordinates, permitting to understand the meaning of the transition to ψ_2 from the equations of field taking place in the formulation of Dirac's equations.

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PHYSICS

ON A CERTAIN PROBLEM IN THE THEORY OF CHARACTERISTICS

By K. NIKOLSKY

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In investigating the characteristics of equations of a quantum particle (equation of Dirac) we meet with the following problem. Let an equation be given which we shall call the Lorentz' equation:

$$\frac{\partial^2 A_\rho}{\partial x^2}+\frac{\partial^2 A_\rho}{\partial y^2}+\frac{\partial^2 A_\rho}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 A_\rho}{\partial t^2}=\lambda^2 A_\rho \quad .(\rho=1,2,3,4). \tag{1}$$

We propose the following problem: let us replace this equation by a system of linear equations with derivatives of the first order with constant coefficients of type (1)

$$\sum_{\sigma=1}^{4}\left(\alpha^4_{\rho\sigma}\frac{\partial\Phi_\sigma}{\partial x_4}+\alpha^x_{\rho\sigma}\frac{\partial\Phi_\sigma}{\partial x}+\alpha^y_{\rho\sigma}\frac{\partial\Phi_\sigma}{\partial y}+\alpha^z_{\rho\sigma}\frac{\partial\Phi_\sigma}{\partial z}-\alpha^m_{\rho\sigma}\lambda\Phi_\sigma\right)=0, \tag{2}$$

where $\alpha^x_{\rho\sigma}$, $\alpha^y_{\rho\sigma}$, $\alpha^z_{\rho\sigma}$, $\alpha^m_{\rho\sigma}$ are constant coefficients and Φ_σ—four, generally complex, functions connected with A_ρ by relations

$$\Phi_\sigma=\sum_{\rho=1}^{4}\left(\beta^4_{\sigma\rho}\frac{\partial A_\rho}{\partial x_4}+\beta^x_{\sigma\rho}\frac{\partial A_\rho}{\partial x}+\beta^y_{\sigma\rho}\frac{\partial A_\rho}{\partial y}+\beta^z_{\sigma\rho}\frac{\partial A_\rho}{\partial z}-\beta^m_{\sigma\rho}\lambda A_\rho\right). \tag{3}$$

Coefficients $\alpha_{\rho\sigma}$ and $\beta_{\sigma\rho}$ should in this case be such as to satisfy for A_ρ equations (1).

Let us introduce matrices

$$(\alpha^4_{\rho\sigma})=\delta_{\rho\sigma} \qquad \beta^4_{\rho\sigma}=-(\delta_{\rho\sigma})$$

where $\delta_{\rho\sigma}=\begin{cases}1 & \text{when } \rho=\sigma\\ 0 & \text{when } \rho\neq\sigma.\end{cases}$

$$\alpha^4=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \qquad \beta^4=-\alpha^4$$

$$\alpha^x=\begin{pmatrix}0&0&0&-1\\0&0&-i&0\\0&+i&0&0\\-1&0&0&0\end{pmatrix} \quad \alpha^y=\begin{pmatrix}0&0&i&0\\0&0&0&-1\\-i&0&0&0\\0&-1&0&0\end{pmatrix} \quad \alpha^z=\begin{pmatrix}0&-i&0&0\\i&0&0&0\\0&0&0&-1\\0&0&-1&0\end{pmatrix} \tag{4}$$

The matrices are Hermitian, since $\alpha^k_{\sigma\rho} = \alpha^k_{\sigma\rho}$, and they satisfy the following conditions:

$$(\alpha^x)^2 = (\alpha^y)^2 = (\alpha^z)^2 = 1, \quad \alpha^x \cdot \alpha^y \cdot \alpha^z = i. \tag{5}$$

$$\left.\begin{aligned} \alpha^x\alpha^y = i\alpha^z \quad \alpha^x\alpha^z + \alpha^z\alpha^x = 0 \\ \alpha^y\alpha^z = i\alpha^x \quad \alpha^y\alpha^z + \alpha^z\alpha^y = 0 \\ \alpha^z\alpha^x = i\alpha^y \quad \alpha^y\alpha^x + \alpha^x\alpha^y = 0 \end{aligned}\right\} \tag{6}$$

As matrices of $\beta^x, \beta^y, \beta^z$ we take $\bar{\alpha}_x, \bar{\alpha}_y, \bar{\alpha}_z$.

Then our problem with $\lambda = 0$ will be a problem of obtaining Maxwell's equations for a vacuum according to given equations of potentials.

Indeed the evident form of equations (2) with $\lambda = 0$ is as follows:

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial\Phi_1}{\partial t} - \frac{\partial\Phi_4}{\partial x} + i\frac{\partial\Phi_2}{\partial y} - i\frac{\partial\Phi_3}{\partial z} = 0 \\ \frac{1}{c}\frac{\partial\Phi_2}{\partial t} - i\frac{\partial\Phi_3}{\partial x} - i\frac{\partial\Phi_4}{\partial y} + i\frac{\partial\Phi_1}{\partial z} = 0 \\ \frac{1}{c}\frac{\partial\Phi_3}{\partial t} + i\frac{\partial\Phi_2}{\partial x} - i\frac{\partial\Phi_1}{\partial y} - \frac{\partial\Phi_4}{\partial z} = 0 \\ \frac{1}{c}\frac{\partial\Phi_4}{\partial t} - \frac{\partial\Phi_1}{\partial x} - \frac{\partial\Phi_2}{\partial y} - \frac{\partial\Phi_3}{\partial z} = 0 \end{aligned}\right\} \tag{7}$$

and of equations (3) with $\lambda = 0$, as follows:

$$\left.\begin{aligned} \Phi_1 = -\frac{1}{c}\frac{\partial A_1}{\partial t} - \frac{\partial A_4}{\partial x} - i\frac{\partial A_3}{\partial y} + i\frac{\partial A_2}{\partial z} \\ \Phi_2 = -\frac{1}{c}\frac{\partial A_2}{\partial t} + i\frac{\partial A_3}{\partial x} - \frac{\partial A_4}{\partial y} - i\frac{\partial A_1}{\partial z} \\ \Phi_3 = -\frac{1}{c}\frac{\partial A_3}{\partial t} - i\frac{\partial A_2}{\partial x} + i\frac{\partial A_1}{\partial y} - \frac{\partial A_4}{\partial z} \\ \Phi_4 = -\frac{1}{c}\frac{\partial A_4}{\partial t} - \frac{\partial A_1}{\partial x} - \frac{\partial A_2}{\partial y} - \frac{\partial A_3}{\partial z} \end{aligned}\right\} \tag{8}$$

Assuming A_ρ to be real [i does not enter (1)] we may divide in (7) and (8) the real and the imaginary parts, assuming

$$\left.\begin{aligned} \Phi_x = \mathfrak{e}_x + iH_x = \left(-\frac{1}{c}\frac{\partial A_1}{\partial t} - \frac{\partial A_4}{\partial x}\right) + i\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_3}{\partial y}\right) \\ \Phi_y = \mathfrak{e}_y + iH_y = \left(-\frac{1}{c}\frac{\partial A_2}{\partial t} - \frac{\partial A_4}{\partial y}\right) + i\left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right) \\ \Phi_z = \mathfrak{e}_z + iH_z = \left(-\frac{1}{c}\frac{\partial A_3}{\partial t} - \frac{\partial A_4}{\partial z}\right) + i\left(\frac{\partial A_1}{\partial y} - \frac{\partial A_2}{\partial x}\right) \end{aligned}\right\} \tag{9}$$

Φ is real according to (7).

Substituting (9) into (7), we obtain

$$\left.\begin{aligned} \frac{1}{c}\frac{\partial\vec{\mathfrak{e}}}{\partial t} - \operatorname{grad}\Phi_4 + \operatorname{rot}\vec{H} = 0, \quad -\frac{1}{c}\frac{\partial\vec{H}}{\partial t} + \operatorname{rot}\vec{\mathfrak{e}} = 0 \\ \operatorname{div}\vec{\mathfrak{e}} = \frac{1}{c}\frac{\partial\Phi_4}{\partial t}, \operatorname{div}\vec{H} = 0, \quad -\Phi_4 = \frac{1}{c}\frac{\partial A_4}{\partial t} + \operatorname{div}\vec{A} \end{aligned}\right\} \tag{10}$$

Matrices (4), (5), (6) afford, in this way, the possibility by linearizing Lorentz' equations to obtain Maxwell's equations.

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CHEMISTRY

RELATION BETWEEN REFRACTION COEFFICIENTS OF HIGHER FATTY ACIDS, TEMPERATURE AND THEIR DEGREE OF UNSATURATEDNESS

By V. J. ANOSOV and G. B. RAVICH

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A universal viscosimetric equation for higher fatty acids C_{18} was proposed in a paper published in Comptes Rendus de l'Acad. Sci. by G. B. Ravich (1).

In this paper we have made it our aim to obtain an analogous refractometric equation for higher fatty acids C_{18} which would show the relation between the refraction coefficient n_D^t, the temperature t and the iodine number I, which is calculated after Hubble's method.

In constructing the n_D^t—I (refraction coefficient—iodine number) isotherm, we have made use of our own experimental data, as well as those of other investigators.

We have measured the refraction coefficients of oleic, linoleic and linolenic acids and of some of their mixtures at 20°, 30° and 40°. The acids used were obtained from the Schering-Kahlbaum firm. The experiments were conducted by V. G. Anosov by means of a refractometer of the Pulfrich-Zeiss type, which was constructed at the Institute for Geodesy and Mapping by G. B. Ravich, with the help of a Zeiss-Loewe refractometer. In performing the experiments, the temperature was kept even, allowing for an error of less than ± 0.1°C (usually ± 0.05° C).

Since the temperature coefficient of fatty acids is approximately 0.0004 (increased a little in approximating), our error in any case was not greater than ± 0.00004. The accuracy of working with the refractometer itself is 0.0001. The resulting data are given in the table.

As the data in table show, the results obtained by Anosov and Ravich, who worked with the same objects, practically coincide in most cases. The values obtained by Holde and Procter are very close to ours. Those obtained by Lund are somewhat lower.

The data given above are interpreted graphically in Fig. 1. From the drawing it is easy to see that the relation n_D^t—I is linear, or very close to it. Practically the parallel straight lines clearly show that within the range 20—40° C the temperature coefficient for the refraction indices of the C_{18} fatty acids varies very little. This gives us grounds to assume the following linear equation for the objects studied, which, as will be shown below, satisfactorily expresses the relation between the refraction coefficient at a given temperature n_D^t and the iodine number I:

$$n_D^t = KI + n_D^{t\,\text{st. ac.}}. \qquad (1)$$

The angular coefficient K of the isotherm turned out to be equal to 0.000106.

We propose in our turn the following equation for finding the refraction coefficient of liquid or supercooled stearic acid, $n_D^{t\ \text{st.ac.}}$:

$$n_D^{t\ \text{st.ac.}} = n_D^{70\ \text{st.}} + 0.00037\,(70 - t). \qquad (2)$$

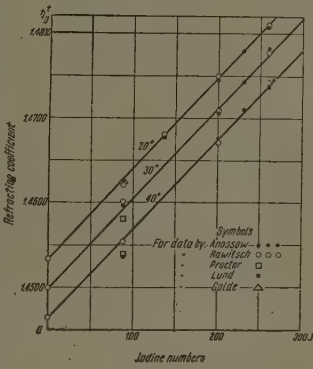
Substituting the value $n_D^{70\ \text{st. ac.}} = 1.435$ into equation (2), we get

$$n_D^{t\ \text{st. ac.}} = 1.435 + 0.00037\,(70 - t). \qquad (2')$$

Substituting the value obtained for $n_D^{t\ \text{st. ac.}}$ from equation (2') into equation (1), we get equation (3) which is a universal refractometric equation for C_{18} fatty acids:

$$n_D^t = 0.000106\,I + 1.435 + 0.00037\,(70 - t). \qquad (3)$$

In the columns numbered II of the table we give the values of the refraction coefficients, as calculated by means of the proposed equation (3). The resultung coincidences may be considered satisfactory. We should make special note of the fact that we have found the refraction coefficient values of stearic acid for the interval of conceivable supercooling, i. e. below its solidifying point (marked in the table by asterisks and in Fig. 1 by double circles), by extrapolating the isotherms $n_D^t - I$ from the region of unsaturated acids (which at these temperatures are liquid) to their points of intersection with the axis, which corresponds to the iodine number $I = 0$. If the ordinates cut off by the isotherms are measured, it is easy on the basis of these measurements to calculate the values of the refraction coefficients of stearic acid at the temperatures in question. The results obtained by means of this method coincide well with the data, calculated by Lund for 40° C and found experimentally by Holde and Pascal(6) for 70°. [According to Holde (4) $n_D^{70} = 1.434$; according to the equation proposed $n_D^{70} = 1.435$].



We believe that this equation is interesting both theoretically and practically not only in fatty acid refractometry. It is obvious that an equation of an analogous type (with other coefficients, of course) may be used in the physico-chemical study of triglycerides and their mixtures, and also in the study and control of hydrogenization (7).

At present we are continuing the experimental and theoretical study of this problem.

C o n c l u s i o n s: 1. The refraction coefficients of the C_{18} fatty acids: oleic, linoleic and linolenic and of some of their mixtures with various iodine numbers, have been measured by means of refractometers of the Pulfrich-Zeiss and Loewe-Zeiss types.

2. It has been shown that the refraction coefficients n_D^t of the C_{18} fatty acids are in linear (or very close to linear) relation to their iodine numbers I. On this basis we propose the following equation:

$$n_D^t = K\,I + n_D^{t\ \text{st. ac.}}$$

where n_D^t is the refraction coefficient of the C_{18} fatty acid at the temperature t; K is the angular coefficient of the $n - I$ isotherms, which equals 0.000106 for the isotherms examined at 20°, 30° and 40° C; $n_D^{t\ \text{st. ac.}}$ is the refraction coefficient of stearic acid at the given temperature.

Acid	Iodine number I	Author	Refraction coefficients n_D^t					
			n_D^{20}		n_D^{30}		n_D^{40}	
			I	II	I	II	I	II
Oleic	89.7	Anosov	1.4630		1.4597		1.45454	
		Ravich	1.4628		1.4600		1.4556	
		Procter[2]	1.4620	1.4630	1.4585	1.4593	1.4546	0.4555
	86.1	Lund[3]	—		—		1.4527	
		Holde[4]	1.463		—		—	
Linoleic	200.1	Anosov	1.47466		1.47102		1.46731	
		Ravich	1.4749		1.4710		1.4671	
	173.3	Lund[3]	—	1.4748	—	1.4711	—	1.4674
		Holde[4]	1.470		—		—	
	181.1	Holde[5]	1.4788		—		—	
Mixture of linoleic and linolenic acids	140.0	Anosov	1.46829		—	—	—	—
		Ravich	1.4683	1.4683	—	—	—	—
Linolenic	256.6	Anosov	1.48081		1.47731		1.47326	
		Ravich	1.4808	1.4808	1.4772	1.4770	1.4738	1.4733
	261.8	Lund[3]	—		—		1.4740	
Mixture of linoleic and linolenic acids	232.6	Anosov	1.47827	—	1.47448	1.47445	1.47100	1.4708
Stearic	0	Lund[3]	—	—	—	—	—	1.4441 *
		Holde[4]	—	1.4537 *	—	1.450 *	—	1.4463 *

3. It has been shown that the refraction coefficients of stearic acid $n_D^{t\,\text{st. ac.}}$ are in linear relation to the temperature, and for the interval 20—70° the temperature coefficient averages 0.00037. This is expressed by the equation:

$$n_D^{t\,\text{st. ac.}} = 1.435 + 0.00037\,(70 - t)$$

where 1.435 is the refraction coefficient of stearic acid at 70°.

4. On the basis of all this we propose the following type of general refractometric equation for C_{18} fatty acids, which expresses the relation between the refraction coefficient n_D^t, the temperature t and the iodine number I.

$$n_D^t = 0.000106\,I + 1.435 + 0.00037\,(70 - t).$$

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CHEMISTRY

ACTION OF WATER ON NATURAL (INDER) BORATES AT VARIOUS TEMPERATURES

By N. M. SELIVANOVA

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1. Action of Water on Natural Calcium Borates. The action of water on natural calcium borates, viz: inyoite $Ca_2B_6O_{11} \cdot 13H_2O$, colemanite $Ca_2B_6O_{11} \cdot 5H_2O$ and pandermite $Ca_8B_{20}O_{38} \cdot 15H_2O$ was studied at temperatures of 20, 50 and 100° C in thermostats with toluol thermoregulators. The temperature in the thermostats varied from —0.2 to +0.2° C.

Perceptible evaporation of the liquid phase at 100° C made it impossible to conduct the experiment in the reaction vessels with mercury stoppers, generally used for these purposes. Therefore these reaction vessels were supplied in addition with condensers.

The weighed portion used for the experiment was 1.5 g. Water—50 cm³ (the water was previously freed from carbon dioxide by repeated distillation).

In all cases the ratio of boric acid to calcium oxide was greater than for the initial borate on the next day after the reaction in the liquid phase had begun. This gives us reason to believe that in this case the calcium borates do not simply dissolve, but hydrolyze.

The compositions of the equilibrous liquid phases produced through the action of water on natural calcium borates are represented in Fig. 1.

It is evident for the pair inyoite—colemanite that the solutions obtained from the hydrolysis of inyoite are supersaturated for analogous colemanite solutions, especially in boric acid, at the temperature mentioned. Hence it is possible for inyoite to turn into colemanite.

The following experiment was performed: a mixture of 2 g inyoite and 1 g colemanite was continuously stirred together in equilibrous inyoite solutions, produced at respective temperatures, in thermostats at 20, 50 and 100° C. At 20° C the matter at the bottom turns entirely into colemanite in 20 days. An analysis of the crystalline product gave: $CaO = 27.15\%$, $B_2O_3 = 50.83\%$, $H_2O = 21.93\%$. Sp. gravity—2.38.

The theoretical composition of colemanite is: $CaO = 27.18\%$; $B_2O_3 = 50.94\%$; $H_2O = 21.88\%$. Sp. gravity—2.42.

The interference lines of the Debye-Scherrer exograph (copper radiation) are analogous to those obtained for natural colemanite.

At 50° C the transformation into colemanite is completed in 15 days. Analysis of the product showed: $CaO = 27.92\%$; $B_2O_3 = 50.83\%$; $H_2O = 21.60\%$.

Sp. gravity—2.38. The exograph of this colemanite coincided with that of the natural mineral.

At 100° C the transformation into colemanite takes 10 days. Analysis showed: CaO=27.60%; B_2O_3=51.01%; H_2O=21. 70%. Sp. gravity—2.38. The exograph coincides with that of natural colemanite.

For the pair inyoite—pandermite all the solutions obtained through the hydrolysis of inyoite are strongly supersaturated for pandermite. In order to transform inyoite entirely into crystalline pandermite the following experiments were performed: a mixture consisting of 1.5 g inyoite and 0.2 g pandermite (as a primer) was added to filtered inyoite solutions obtained at 20, 50 and 100° C two days after hydrolysis had set in.

At 20° C the transformation of inyoite into pandermite took 15 days. Analysis showed: CaO=31.95%; B_2O_3=49.33%; H_2O=18.88% (total

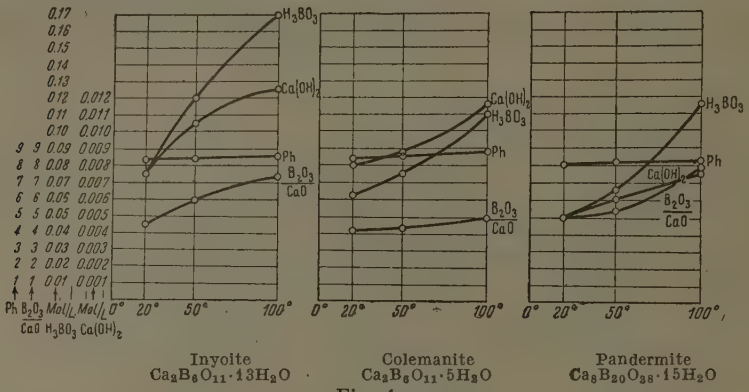


Fig. 1.

100.16%); B_2O_3/CaO=1.25. Sp. gravity—2.47. The exograph coincides with that of natural pandermite, and the microphotographs also.

At 50° C the transformation into pandermite took 12 days. Analysis: CaO=31.83%; B_2O_3=49.21%; H_2O=19.19%; B_2O_3/CaO=1.24. Sp. gravity—2.47. The exograph and the microphotograph coincide with those of natural pandermite.

At 100° the formation of pandermite was completed in 8 days. Analysis: CaO=32.03%; B_2O_3=49.07%; H_2O=19.07%; B_2O_3/CaO=1.25. Sp. gravity—2.47. The exograph and microphotograph are analogous with those of natural pandermite. The indices of refraction are: N_g=1.594; N_p=1.576. For natural pandermite: N_g=1.594 and N_p=1.572.

By acting on inyoite with water in the absence of priming we were successful in obtaining a product, whose composition and interference lines of the Debye-Scherrer exograph showed it to be pandermite.

The specific gravity of this product is 1.83. Under the microscope is to be seen an isotropic mass with a refraction index of $1.598>N_m<1.570$.

The product is apparently an allotropic and metastable modification of pandermite; after being 8 months in a solution of boric acid and calcium oxide, it changed completely into tiny rhombic crystals, whose optical constants were identical with those of natural pandermite.

For the pair colemanite—pandermite solutions obtained through the hydrolysis of colemanite were supersaturated for pandermite in all cases except at 100° C. At 100° C a solution of pandermite in boric acid is supersaturated for colemanite.

The transformation of colemanite into pandermite is a very slow process. The following experiments were performed: a mixture of 1 g colemanite and 1 g pandermite was rotated at 20° C in a colemanite solution, produced on the second day after hydrolysis at 20° C. After 45 days a product of the following composition was obtained: $CaO = 32.01\%$; $B_2O_3 = 48.97\%$; $H_2O = 18.93\%$; $B_2O_3/CaO = 1.24$. Sp. gravity—2.47.

The exograph is identical with that of natural pandermite.

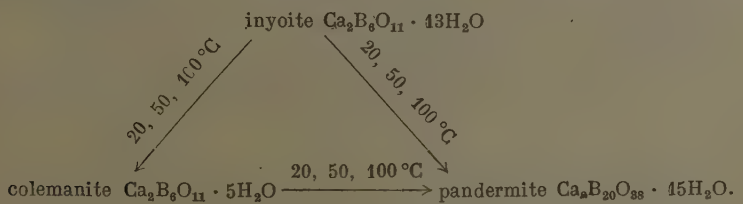
An experiment was performed under analogous conditions at 50° C. The transformation into pandermite was observed only after 40 days. Analysis: $CaO = 31.92\%$; $B_2O_3 = 48.91\%$; H_2O—18.97%; $B_2O_3/CaO = 1.25$. Sp. gravity—2.47.

At 100° C the reaction took 50 days. We were successful in attaining complete transformation into pandermite. An analysis of the product showed: $CaO = 29.48\%$; $B_2O_3 = 50.03\%$; $H_2O = 20.41\%$; $B_2O_3/CaO = 1.33$.

It is obvious that the presence of an excess of boric acid when pandermite hydrolyzes (Fig. 1) hampered further decomposition of the colemanite. But it must be noted that the exograph of the product with the ratio $B_2O_3/CaO = 1.33$ is analogous to that of natural pandermite.

In general an X-ray examination of the hydrolysis products of inyoite and colemanite with the ratio B_2O_3/CaO = from 1.33 to 0.10 showed that they give the same interference lines as natural pandermite on the Debye-Sherrer exographs.

Thus, the following transformation scheme is established for natural calcium borates:



2. Action of Water on Hydroboracite and Transformation of Hydroboracite into Asharite. Preliminary experiments have shown that when water acts upon hydroboracite $CaMgB_6O_{11}.6H_2O$, the calcium ions are washed out more intensively than magnesium ions and at the same time boric acid passes into the liquid phase more intensively than the bases. This traces a path for the transformation of hydroboracite into asharite.

As a result of very prolonged action of water on hydroboracite at 20 and 50° C, an amorphous product was obtained, which by its composition turned out to be asharite with two molecules of water.

By acting with water at 100° C we were successful in transforming hydroboracite into asharite incompletely in 5 months. Data of an analysis of the product: $B_2O_3/MgO = 0.51$; $MgO = 45.72\%$; $B_2O_3 = 41.04\%$; $H_2O = 13.34\%$. Sp. gravity—2.58.

The theoretical composition of asharite is: $B_2O_3/MgO = 0.50$; $MgO = 47.92\%$; $B_2O_3 = 41.38\%$; $H_2O = 10.70\%$. Sp. gravity of Inder asharite—2.69.

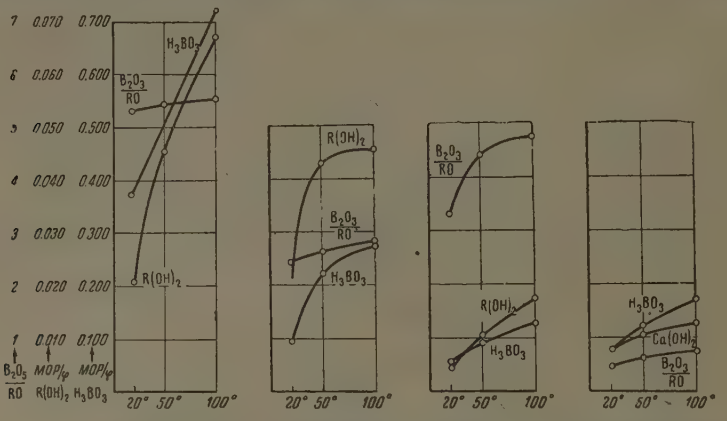
Tiny needle-like crystals with the optical constants $N_g = 1.647$, $N_p = 1.576$ are discernible in the amorphous mass within the field of the polarization microscope. The refraction indices of Inder asharite are: $N_g = 1.647$; $N_p = 1.576$.

The temperature stops on the heating curve of the product and the interference lines on the Debye-Sherrer exograph (copper radiation) turned out

to be identical with those of natural asharite. The product is slightly soluble in dilute hydrochloric acid.

We were unsuccessful in attaining complete transformation of the amorphous mass into crystalline asharite.

3. Transformation of Magnesium Borates into Double and Calcium Borates. In comparing the composition of the liquid phases which are produced through the action of water on kaliborite, boronatrocalcite, hydroboracite and inyoite, we found that those of kaliborite are greatly supersaturated for boronatrocalcite, hydroboracite and inyoite (Fig. 2). Therefore the transformation kaliborite→boronatrocalcite



Kaliborite $KMg_2B_{11}O_{19}\cdot 9H_2O$ Boronatrocalcite $NaCaB_5O_8\cdot 8H_2O$ Hydroboracite $CaMgB_6O_{11}\cdot 6H_2O$ Inyoite $Ca_2B_6O_{11}\cdot 13H_2O$

Fig. 2.

should be attainable in the presence of salt and gypsum, and also the transformation kaliborite→hydroboracite→inyoite in the presence of gypsum.

The synthesis of boronatrocalcite from kaliborite in the presence of common salt and gypsum was attained by Vant Hoff at 83° C and by A. V. Nikolaiev and A. G. Čelišćeva at 25° C. When kaliborite and gypsum are mixed together in water at 20° C, the solid phase turned out to be composed of two forms of crystals after 10 months: 1) large, well-formed crystals of hydroboracite with the refraction indices: $N_g=1.571$; $N_m=1.533$; $N_p=1.522$, which coincide well with the optical constants of Inder hydroboracite: $N_g=1.572$; $N_m=1.535$; $N_p=1.522$.

2) well-formed crystals of inyoite with the refraction indices: $N_g=1.516$; $N_m=1.505$, $N_p=1.492$.

The refraction indices of Inder inyoite are: $N_g=1.519$; $N_m=1.505$; $N_p=1.480$.

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CHEMISTRY

OXIDATION-REDUCTION POTENTIALS AS APPLIED TO THE STUDY OF THE PARAGENESIS OF MINERALS

By V. V. SČERBINA

(Communicated by A. E. Fersman, Member of the Academy, 19.I. 1939)

The application of the theory of oxidation-reduction processes in the study of mineral paragenesis makes it possible to explain the relation between the mineral composition and the oxidation-reduction potential of the medium.

Thus, although we can often substitute the ratio Ca : Mg for CaO : MgO, for Fe and Mn one atomic ratio may refer to three different mineral combinations: siderite—rhodochrosite, limonite—rhodochrosite, and limonite—pyrolusite, depending on the degree of oxidation.

The following elements are found in various stages of oxidation in the natural minerals of the surface zone:

Fe (2, 3); Mn(2, 3, 4); V(3, 4, 5); Ti(3?, 4); Cr(3, 6); Co(2,3); Ni(2,3); Mo(5, 6); U(4,6); N(−3, +5); I(−1, +5); Cu(0,1,2); Pb(2, 4); As and Sb (−3, 0, +3, +5); Se and Te (+4, +6).

The reduction-oxidation potential values of the ions composing the system determine the direction and intensity of these processes. The following are the values for the above elements, arranged in ascending order, expressed in volts and referring to a $1n$ solution at 18°.

Table 1

$Fe(OH)_2 \rightarrow Fe(OH)_3$		$= -0.65$	Mo^{5+}	$\rightarrow Mo^{6+}$	$= +0.78$
Ti^{3+}	$\rightarrow Ti^{4+}$	$= -0.04$	I^{1-}	$\rightarrow (IO_3)^{1-}$	$= +1.08$
Cu^{1+}	$\rightarrow Cu^{2+}$	$= +0.167$	V^{4+}	$\rightarrow V^{5+}$	$= +1.24$
$(SO_3)^{2-}$	$\rightarrow (SO_4)^{2-}$	$= +0.22$	Cr^{3+}	$\rightarrow Cr^{6+}$	$= +1.30$
Cu^{2}	$\rightarrow Cu^{2+}$	$= +0.345$	Mn^{2+}	$\rightarrow (MnO_2)_{solid}$	$= +1.35$
V^{3+}	$\rightarrow V^{4+}$	$= +0.40$	Pb^{2+}	$\rightarrow (PbO_2)_{solid}$	$= +1.44$
U^{4+}	$\rightarrow U^{6+}$	$= +0.41$	MnO_2	$\rightarrow (MnO_4)^{1-}$	$= +1.59$
$Ni(OH)_2 \rightarrow Ni(OH)_3$		$= +0.49$	Co^{2+}	$\rightarrow Co^{3+}$	$= +1.79$
Fe^{2+}	$\rightarrow Fe^{3+}$	$= +0.75$	Pb^{2+}	$\rightarrow Pb^{4+}$	$= +1.80$

Any one of these reduction-oxidation pairs is an oxidizer with regard to those situated above it, and a reducer with regard to those situated below it. The greater the difference of potentials, the more intensive the reaction. As the table shows, potentials in the ion state differ noticeably from potentials of ion-solid hydroxide or hydro-protoxide—hydroxide. The potential value depends on the concentration, which follows from the well-known formula:

$$E_1 = E_0 + \frac{0.058}{n} \log \frac{C_1}{C_0}$$

(E_1 and E_0 are potentials at the given and normal concentrations respectively, C_1 and C_0 are the concentrations of the given and the normal solutions). Thus, for instance, the direction of the reaction $Fe^{2+} + Mo^{6+} \leftrightarrows Fe^{3+} + Mo^{5+}$ is determined by the concentrations of the iron and molybdenum compounds; at low Fe^{2+}-ion concentration and high concentration of the $[Mo^{6+}O_4]^{2-}$ ion, the mineral ilzemanite $Mo_2O_5.3MoO_3.6H_2O$ is formed in the presence of ferric compounds, on the other hand, low concentrations of hexavalent molybdenum compounds are stable in the presence of large quantities of ferrous iron, which explains the occurrence of wulfenite ($PbMoO_4$) crystals on siderite (in the Przhibram layer, for instance). The equilibrium point of this reaction is, when the concentration ratio of the ions Fe : Mo = 3.3 : 1.

From Table 1 it follows that the potentials of dissolved ions and of hydroxides differ considerably, especially in alkaline solution. The potentials given for $Co^{2+} \rightarrow Co^{3+}$ and for $Pb^{2+} \rightarrow Pb^{4+}$, which are higher (about 1.8) than the potential of the formation of permanganate (about 1.6) which does not occur in nature, make us assume that steinirite $Co_2O_3 . 2H_2O$ and plattnerite PbO_2 formed from ions dissolved not in acid solution (although solid PbO_2 begins to form when the potential is + 1.44 volts), but in alkaline solution, in particular, the potential value for the reaction $[Pb^{2+}O_2]^{2-} \rightarrow [Pb^{4+}O_3]^{2-}$ is only + 0.23 volts.

The formation of PbO_2 in alkaline solution is proved qualitatively by means of the following reactions:

in acid solution: $PbO_2 + Mn^{2+} \rightarrow Pb^{2+} + [MnO_4]^{1-}$,

in alkaline solution: $KMnO_4 + Pb(OK)_2 \rightarrow PbO_2 + MnO_2$, i. e. the direction of reaction is opposite and in the second case the compounds formed are known in the form of natural minerals. Finally, the inert anions also display some influence over the potential value. It should be noted that reduction-oxidation potentials depend on the temperature as well. The guide in the paragenesis of ous-ic minerals is iron, which is the most widely distributed of all the elements found in various stages of oxidation. Table 2 gives the concurrence and the impossibility of occurrence of ions of other elements with iron ions (valencies given in brackets).

Table 2

In the presence of	May occur	Do not occur
Fe^{2+}	Ti(3,4); V(3,4); Cr(3); Mo(5,6); Mn(2); Co(2); Ni(2); U(4,6); W(6); Pb(2)	V(5); Cr(6); Mn(4); Co(3); Ni(3); $[IO_3]^{1-}$; $[SeO_3]^{2-}$; $(NO_3)^{1-}$; Pb(4)
Fe^{3+}	Ti(4); V(4,5); Cr(3,6); Mo(5,6); Mn(2,4); Co(2,3); Ni(2,3); U(6); $(IO_3)^{1-}$; $(NO_3)^{1-}$; $(SeO_3)^{2-}$; Pb(2,4)	Ti(3); V(3); U(4)

Analogous tables can be given for any other ion.

The occurrence of reduction-oxidation reactions can be drawn up in the following scheme (Table 3).

The elements are arranged in order of ascending potentials; the highest form of oxidation is in the horizontal row, the lowest, in the vertical row. Opposite the ions of like elements are given their potential values; the oxidizing strength increases from the bottom upwards; in the upper right corner the reaction takes place (the reactions marked with a sign are those checked

Table 3

	Ti^{4+}	V^{4+}	U^{6+}	Fe^{3+}	V^{5+}	Cr^{6+}	$(MnO_2)_{solid}$	$(PbO_2)^2$	Co^{3+}
Ti^{3+}	−0.04	?	?	+	+	+	+	+	+
V^{3+}	?	+0.40	?	+	+	+	+	+	+
U^{4+}	?	?	+0.41	+	+	+	+	+	+
Fe^{2+}	—	—	—	+0.75	+	+	+	+	+
V^{4+}	—	—	—	—	+1.24	—	?	+	+
Cr^{3+}	—	—	—	—	—	+1.30	?	+	+
Mn^{2+}	—	—	—	—	—	—	+1.35	+	+
Pb^{2+}	—	—	—	—	—	—	—	+1.44	—
Co^{2+}	—	—	—	—	—	—	—	?	+1.79

by the author); their intensity increases from the diagonal with numbers towards the right upper corner. In the lower left corner there is no reaction (—sign; the? sign denotes those which have not yet been ascertained). The direction from the upper left to the lower right corresponds to definite degrees of oxidation, which make it possible to build a conception of the oxidation facies of hypergeneic minerals and to classify them.

I. Ferrous iron facies (no ferric iron). The ions present (Table 2) are those which theoretically can exist together in the form of the following minerals: siderite $FeCO_3$; rhodochrosite $MnCO_3$ (practically some of these compounds will be dissolved isomorphically in $FeCO_3$), stichtite $6MgO \cdot Cr_2O_3 \cdot CO_2 \cdot 12H_2O$, uranothallite $Ca_2U(CO_3)_4 \cdot 10H_2O$ and compounds of Mo^{5+} and V^{5+} or V^{+}. Minerals, contaning ions which ferrous iron, trivalent vanadium or tetravalent uranium reduce, cannot be present (trivalent vanadium and tetravalent uranium reduce ferric iron to ferrous).

II. Ferrous-ferric iron facies. The following ions may be present: Mn(2); Ti(4); V(4); Cr(3); Mo(6); Pb(2); Co(2); Ni(2); U(6). The following ions which are reduced by ferrous iron cannot be present: Mn(4); V(5); Cr(6); Pb(4); Co(3); Ni(3); $[IO_3]^{1-}$; $[NO_3]^{1-}$; $(SeO_3)^{2-}$, nor the following which are oxidized by trivalent iron: Ti(3); V(3); U(4) (for the position of the facies in the oxidizing series see Table 3).

III. Ferric iron facies. No ferrous iron and no ions which are oxidized by ferric iron. As the oxidation level rises, other elements also gradually pass into the higher oxidation stages.

IV. Chromate and vanadate facies. Only Mo(6) (wulfenite) present, which associates with vanadite and crocoite. The potential of the medium has a value of about +1.2—+1.3 volts, which fixes the oxidation of some ions and the reduction of others (Pb^{4+}, Co^{3+}).

V. Facies of higher natural oxides. Potential over +1.35. All the manganese is in the dioxide form; there is no Mn^{2+}, Cr^{3+}, V^{4+}; PbO_2 (plattnerite), $Co_2O_3.2H_2O$ (steinirite) and $Ni_2O_3 . nH_2O$ may form.

Special nitrate facies. No lower than IV, possibly higher than V. Example—the Chili saltpetre beds which contain $[NO^3]^{1-}$, $[IO_3]^{1-}$, $[CrO_4]^{2-}$ (their potentials: +1.24, +1.08, +1.30).

The minerals which form the top limit of this facies are unknown. According to A. E. Fersman, ions of this facies may be distinguished by their energy constants which are very low (1).

A detailed study of the paragenesis of hypergenic minerals [see, for instance, S. S. Smirnov's paper (2)] makes it possible to broaden these conceptions and make them more detailed.

Conclusions

The theoretic scheme described serves to explain a number of practical problems and permits of making a few prognoses:

1. It explains paragenetic association in the cap-rock zones of sulphide beds and also the conditions, under which silver and gold are transported (their potentials are +0.81 and +1.38).

2. It shows the conditions, under which vanadium beds of the Suleiman-Sai, Ottavi etc. types are formed, since vanadium concentrates in the quinquevalent oxidation form, and at the same time it explains why vanadium is not concentrated in the weathering zone of the Ural titanomagnetites and the rocks which hold them and which contain also vanadium (^{3}) and ferrous iron.

3. It explains the separation of Co and Ni observed in the hypergenesis zone by I. I. Ginsburg. Cobalt passes into the trivalent state considerably easier than nickel.

4. A detailed study of the facies MnO_2, PbO_2, Co_2O_3 will give grounds to draw a number of interesting theoretical and practical conclusions on the concentrations of elements found in manganese ores and on the forms, which there are found in. The problem of natural tetravalent cerium minerals and of the conditions, under which they are formed, also belongs here.

5. It explains the genesis of ilzemanite, and as to chromium it explains why stichtite and volkonskoite are formed instead of crocoite after the weathering of chromite beds (where there is an excess of ferrous iron), i. e. the data given here explain the paragenesis of hypergenic minerals which are formed by ions in various stages of oxidation.

These studies are to be extended to high-temperature minerals as well.

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MINERALOGY

ON DEMANTOID FROM THE ULTRA-BASIC ROCKS OF AZERBAIDJAN

By MIR-ALI KASHKAI

(Communicated by A. E. Fersman, Member of the Academy, 5.II. 1939)

Demantoid in the ultra-basic complex of Azerbaidjan was for the first time discovered by me at the source of the small river Kashka-Bulakh on the slopes of Mt. Shah-Dag (Minor Caucasus), where serpentinized dunite ([1,2]) has the greatest propagation.

The dunites among which demantoids have been discovered can be macroscopically easily identified by their peculiar bottle-green colour and their pale-green weathering crust. In the regions where intensive weathering of hyperbasites and in particular dunites is observable, it is often possible to find a friable snow-white product consisting in the main of calcium carbonate and magnesium hydrocarbonate. In disseminated form serpentinized dunite contains small grains of chromespinelide, coming in certain places up to 3—4 % of the main mineral. Let us note here that dunites are, as has been ascertained, genetically associated with the larger accumulations of chromite in the ultra-basic zone of Azerbaidjan([1]).

Accumulations of demantoid among hyperbasites are observable at definite sites*. It occurs in strongly crumpled fissured rock in the form of an emerald-green scale, or as thin veins in strongly metamorphozed dunites of pale-green colouring. In some places grains of demantoid having an isometrical form and a diameter of 1 mm to 0.5 cm are also formed on the walls of fissures, or of lens-shaped cavities, or else on the surface of serpentinized dunite where the above-mentioned weathering crust fails to be observed.

The metamorphozed rock possesses a characteristic oily lustre. It is intersected by a number of parallel fissures. The rock consists, in the main, of metaxite and a certain amount of carbonate (magnesite), demantoid and chromespinelide. Metaxite is of pale-green or yellowish-green colour. It somewhat resembles talc by its macroscopic appearance, but differs from it on account of a denser woody texture. The double refraction of the metaxite determined by means of a Bereck compensator was found to be $N_g - N_p = 0.005 - 0.006$. Elongation is positive. Refraction coefficient $N_m = 1.553 + 0.001$.

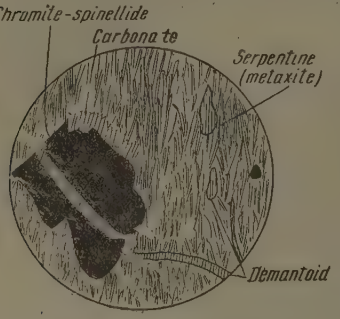
Magnesite forms small flakes which in the strongly metamorphozed rock accumulate around the grains of chromespinelides. The magnesite flakes

* In the summer of 1938 demantoid having the form of scales was found by G. Efendiev in the locality of Churman (Azerbaidjan SSR).

cutting through the rock in the form of thin veins become intergrown with the filaments of metaxite and on the whole tend to run in a perpendicular direction to the walls of their small veins.

Chromespinelides, dotted or as aggregates on account of the relative impoverishment by the molecule $FeCr_2O_4$ (the content of Al_2O_3 increasing in return), are semi-transparent and shine through with a bloody-red glow.

Demantoid, forming grains of rounded and often isometrical shape, pierces not only through the rock with its thin veins, but even interlaces separate minerals, as can be seen from the illustration (figure). The mineral is transparent and has an emerald-green colour. Viewed through the microscope, it retains its colour. It is optically isotropic. No anomalies are observable. The coefficient of refraction determined for the corresponding alloys was found for demantoids, formed both in the fissures and the voids of serpentinized dunites, as well as in the weathering crust, to be equal to 1.88 ± 0.03 (which is somewhat higher than for ordinary demantoids).



According to some observed faces, the shape of the mineral approaches that of a rhombic dodecahedron. As to its macroscopic appearance, the mineral in question bears full analogy with the demantoid from Bobrovka.

For chemical characteristics of the demantoid from Mt. Shah-Dag we give here two analyses (table) performed in the Geochemical laboratory of the Azerbaidjan branch of the Academy of Sciences of the USSR by G. Efendiev (analysis № 1) and F. Vekilova (analysis № 2), as well as five analyses from the works by I. Waller (7), Rammelsberg (5), Seebach (6) and A. Lösch(3).

№ of analysis / Components in % by weight		1	2	3	4	5	6	7
	Demantoid from Mt. Shah-Dag			Accord. to Waller (7)	Accord. to Rammelsberg(5)	Accord. to Seebach (6)	According to Lösch (3)	
	emerald-green crystals						grass-green colour	emerald green colour
SiO_2	34.52	0.575*	32.09	35.69	35.44	35.44	35.56	35.50
TiO_2	not det.	—	nil	—	—	—	—	—
Al_2O_3	nil	—	0.33	0.19	—	1.63	0.57	0.70
Cr_2O_3	not det.	—	—	—	—	1.36	—	—
Fe_2O_3	31.33	0.196	25.72	29.96	32.85	28.80	30.80	31.51
FeO	—	—	1.42	1.25	—	0.52	0.64	—
MgO	0.72	0.018	11.92	0.08	0.20	0.16	0.16	0.21
CaO	33.42	0.596	12.98	32.33	32.85	32.18	33.05	32.90
K_2O	not det.	—	not. det.	0.25				
Na_2O	» »	—	» »	0.63				
H_2O, −110°	» »	—	0.37					
H_2O, +110°	» »	—	6.02					
MnO	« »	—	nil			0.29		
NiO	» »	—	traces					
Total:	99.99			100.38	101.34	100.38	100.78	100.82

* Reduced to terms of molecular amounts.

Specific gravity at a temperature of 20.5° for the demantoid from Mt. Shah-Dag=3.78 and for the demantoid from Sysertsky region=3.838.

Analysis 1 — Emerald-green small crystals of demantoid from Mt. Shah-Dag found at the source of river Kashka-Bulakh.

Analysis 2 — Scales and veins of demantoid scraped off for analysis purpose from the surface of a metamorphozed dunite: Mt. Shah-Dag, the same spot.

Analyses 3, 4, 5, 6, 7 — Demantoid from v. Poldneva of Sysertsky region (Ural).

After conversion of analysis № 1 we obtain the following composition of the demantoid: $Ca_3Fe_2(SiO_4)_3$. Molecular oxide ratio $RO : R_2O_3 : SiO_2 = 3 : 1 : 3$.

Analysis № 2 reduced to terms of mineralogical composition fails to correspond to the pure demantoid. A sample of the latter contains an admixture of metaxite which is confirmed by microscopic examinations.

In conclusion, if we want to consider the origin of the demantoid belonging to the ultra-basic complex of the Transcaucasia, it should be stated, that accumulations of this mineral have been observed in much crumpled, metamorphozed serpentinized dunites in the form of very thin penetrating small veins occurring both in the rock and on the surface of the dunites. The formation of the demantoid was no doubt furthered by metasomatic processes associated with postvolcanic activity on Mt. Shah-Dag. The carried-in material was chiefly calcium of the carbonic thermae.

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MINERALOGY

GENESIS OF MONTMORILLONITE AND KAOLINITE AND CONDITIONS OF THEIR JOINT OCCURRENCE IN THE COLLOIDS OF SOILS AND CLAYS

By I. D. SEDLETZKY

(Communicated by W. I. Vernadsky, Member of the Academy, 24.I. 1939)

1. Extensive X-ray and thermal investigations of the colloids of soils and clays of the Soviet Union, conducted within the last four years by the Soil Institute of the Academy of Sciences of the USSR, have shown that soil colloids have in almost all cases been formed by crystalline argillaceous minerals. The predominant components are:

1) Minerals of the kaolin group (kaolinite and dickite).

2) Montmorillonite and beidelite.

Minerals of the group of nontronites and halloysites are to be found much less frequently. All the minerals, discovered in the colloids of soils, contain a kaolin nucleus and belong to the class of alumo-acids and oxy-alumo-acids. Thus, the keen sagacity of W. I. Vernadsky is justified to the effect that the predominant compounds in the weathering crust are alumo-acids and oxy-alumo-acids, the «kaolin nucleus» being at the back of their texture (1).

2. A thorough-going examination of the mineralogical composition of soil colloids has proved the fact that the composition of colloid minerals fails to be always constant, but undergoes sharp changes according to the types of soil and the respective genetic horizons.

In soil colloids of the alkaline series, i. e. in solonetz and solonetz-like soils, there are always to be discovered only the following minerals: montmorillonite, beidelite and sericite, whereas in soil colloids of the acid series, i. e. in podzols and podzol-like soils, the prevailing minerals are those of the kaolin group. In 1936 by way of suggesting a working hypothesis, we advanced the statement that such minerals as montmorillonite, beidelite and sericite, as well as some others, are formed only in soils of the alkaline series, while in acid soils (in podzols and podzol-like soils) there arise minerals of the kaolin group (2). In soils with a neutral medium, i. e. in soils occupying an intermediate position between alkaline (solonetz) and acid (podzolaceous) soils, there should arise either mixed or specific minerals*.

All these minerals are thus produced in soils and they are therefore soil formations, or soil minerals (pedolites).

3. Within the last two years extensive work has been accomplished tending to impart greater precision to the hypothesis on the dependence of

* Our hypothesis is in perfect agreement with the experimental data obtained by Nöll (3).

the composition of pedolites upon the type of soil formation and also on the genesis of montmorillonite and kaolinite. Solonetz soils from the following regions were subjected to investigation: Trans-Volga, Caspian plain, Middle Asia, Tchernigov district etc., and also podzol soils from the Kalinin district, the Moscow district, Kamchatka, etc.

4. In solonetz soils in all cases colloids contained montmorillonites or beidelites. It was observed that in soda-solonetz montmorillonite prevailed in the colloids throughout the profile (from 0 to 200 cm and lower), whereas in sulphate-chloride solonetz it was only in the two upper horizons (from 0 to 25 cm) that the colloids contained beidelite*. We find an explanation of this in the genesis of the soils in question. The sulphate-chloride solonetz (No 17) according to the classification of V. Kovda ([5]) represents an initial stage, while the soda solonetz (No 26) forms a more advanced one.

Due to the subsidence of ground waters a desalinization of salt-marsches takes place. Easily soluble salts are driven away from the upper horizons, whereupon there appears a solonetz horizon. The sulphate-chloride solonetz represents an initial stage in solonetz formation. Very characteristic is the

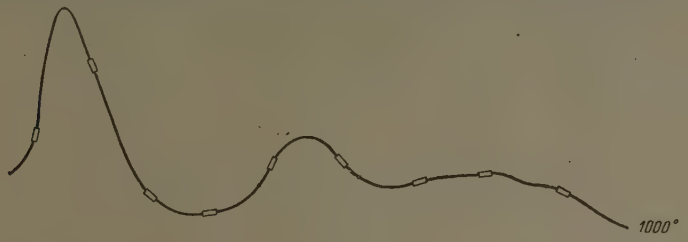

Fig. 1.

fact of boiling on the solonetz horizon at 10—15 cm and the occurrence of gypsum at 20—25 cm. The form of the segregation of carbonates as worm-like veins running on top of the horizon of gypsum segregation of the same shape leads us to suppose that the stated isolation of carbonates in solonetz horizons is a kind of pseudomorphosis over the gypsum that formerly existed there. The desalinization and salinization process is accompanied by the formation of soda and involves the reaction of the formation anew of carbonates of calcium and their accumulation in place of the former gypsum horizons**.

$$CaSO_4 + Na_2CO_3 \rightleftarrows CaCO_3 + Na_2SO_4.$$

The almost total absence of soda in water extractions is accounted for by the fact that its formation is impeded by the presence in these solonetz formations of sodium salts, available almost from the very surface, and preventing the absorbed sodium to dissociate. Another circumstance which appears to play a certain part here is that every new portion of soda is spent on the formation of $CaCO_3$ from gypsum. Hence it follows with perfect obviousness that a weakly alkaline reaction in a sulphate-chloride solonetz will take place only in the upper horizons, i. e. above the horizon of gypsum where soda is at times formed. It is precisely in these horizons that beidelite had been discovered (Fig. 1); below the gypsum horizon which impedes the admis-

* Montmorillonite is not easily distinguishable by means of X-ray investigation from beidelite. A clean-cut distinction between these two minerals is observable in the curves of heating according to the Le Chatelier differential method. In conformity with Orsel's opinion ([4]) beidelite and montmorillonite have a common first endothermal jog at 120—150°, but differing second jogs. Beidelite gives second endothermal jog at 550°, while montmorillonite at 700—800°. Unlike kaolinite, beidelite has no exothermal jog at 950°. In the case in hand, beidelite gives an exograph clearly distinct from that of kaolinite.

** According to Kovda's scheme ([5]).

sion of soda from above, there predominates a weakly acid medium while beidelite and montmorillonite are absent. With a further lowering of the level of ground waters and with the development of the process of salinization a more intensive formation of soda takes place in the upper horizons, while a gradual disappearance of the gypsum horizon becomes apparent. In the soda-solonetz we have no gypsum down to a depth of 200 cm and accordingly soda is able to percolate freely from above, coming down to a great depth and setting up a strongly alkaline reaction along the whole soil profile. The water extraction and soil waters have a pH>9.5. In this way, highly favourable conditions for the formation of the montmorillonite are available here, and indeed we observe that throughout the depth of the profile down to 200 cm the colloids of the soda-solonetz actually contain montmorillonite (Fig. 2). Thus, we see from these examples that the formation of montmorillonite* and beidelite does practically occur under the alkaline conditions of solonetz soils. The solonetz process produces montmorillonite and beidelite.

5. Certain podzol soils examined by us have shown the presence of groups of kaolinite and montmorillonite in mineral colloids (minerals of the kaoline group prevailed).

This circumstance would as it were frustrate our hypothesis. But here our attention was drawn to the fact that in the lowermost horizons kaolinite

Fig. 2.

is apt to vanish away. Further investigation of the deeper layers of podzol and red soils has shown the absence of minerals of the kaolin group here. In all cases investigated by us, the soils were formed on clays which already contained montmorillonite**. Thus, montmorillonite is a mineral of the mother rock (in the present case clays) upon which the soils were formed.

The podzol-formation process—acid in its essence—involved a partial destruction of the montmorillonite of clay colloids (these clays might have been formed under alkaline conditions). Simultaneously with the destruction of the montmorillonite, the formation of new minerals of the kaolin group also took place under the influence of the acid podzol-forming process.

Thus, the montmorillonite is a mineral of the mother soil-forming rock***, whereas kaolinite owes its appearance to the soil-forming process. The podzol-forming process produces kaolin and quartz. We obtain evidential support for this contention also from investigations of separate layers of the weathering of the spherical separations from Tzikhis-Dziri. In the red soil weathering crust spherical separations are observable at various depths. In result of weathering, the mass of spherical separations breaks up into a number of layers, among which the most weathered is the outermost.

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* W. Nöll [(3), Chemie der Erde, 11, 294, 1937] believes that montmorillonite is formed under hydrothermal conditions.

** Unfortunately we have at our disposal no podzol soils formed on eruptive rocks.

*** Distinction should, therefore, be made in colloidal soils between soil minerals, that is those which were formed in soils (pedolites), and minerals of the mother rocks having passed over into the soil and having not had time to disintegrate (detrites and petrolites).

Towards the centre lie less weathered layers and in the very centre is to be found the unchanged rock (porfirite). The spherical separation examined by us had 18 layers which possessed the following pH:

pH =	6.22	6.31	6.81	6.31	7.00	6.65	6.53	6.17	7.86	8.42
No of layer ...	1	2	3	5	7	9	11	13	16	18

In the initial stages of weathering an alkali medium is available, which in later stages is replaced by a slightly acid and acid medium. It stands to reason that the change in the character of the reaction is connected with the carrying away of bases. Our investigations of the 18th layer with an alkaline reaction and of the 5th layer with an acid reaction have brought out the fact that the colloids of the 18th layer are represented by montmorillonite, while the curve of heating for the colloids of the 5th layer shows the presence of small amounts of kaolinite (Fig. 3) along with montmorillonite. The availability of kaolinite is evidenced by an endothermic jog at 550° and an exothermic one at 880°. Thus, montmorillonite is formed in the first stages of weathering under alkali conditions too. With the onset of acid conditions kaolinite is formed.

Consequently, montmorillonite and kaolinite are not formed under the same physico-chemical conditions, as this was held by Orsel and Cailler (6).

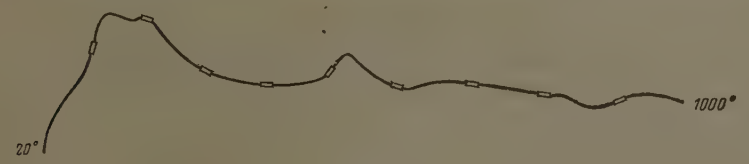

Fig. 3.

The joint occurrence of montmorillonite and kaolinite testifies to the presence of acid conditions. However, the possibility of a reverse process is not excluded, i. e. the substitution of alkali conditions for acid ones in result of the salinization of kaolin clays.

6. It follows from our above described investigations that montmorillonite is formed under alkaline conditions, whereas kaolinite is formed in an acid medium. A prolonged period of prevailing alkali and acid conditions should necessarily lead to the formation of montmorillonite or kaolin clays. And, in fact, kaolin clays are actually to be found in such localities where acid conditions prevail. It has long since been observed that the strata underlying coal measures are kaolin clays [Harrassowitz (7)]. Špalek (8) states that the kaolin of Moravia was formed in the Miocene due to the effect of organic acids from peat bogs. Such a close relationship existing between kaolin clays and coals becomes particularly clear, if we take into consideration the fact that humic acids are apt to arise in the process of the formation of coals, which, penetrating into the depth of the rocks underbedding the coal measures, produce a strong change in them. This change is towards the formation of kaolin minerals, stable under acid conditions.

We have studied in co-operation with S. Yussoupova (1938) a few clays from the Uzbekistan deposits under Jurassic coal beds. The colloids of these clays contain kaolinite and quartz. The presence of quartz together with kaolinite speaks in favour of the possible breaking down of montmorillonite with the formation of kaolinite and quartz, being the result of the carrying out of sesquioxides in an acid medium (mainly aluminium).

In contradistinction to kaolin clays, montmorillonite clays are mainly located in regions of the accumulation of soluble salts. The clays of the Syrt, extending throughout the Caspian lowland, have a montmorillonite composition of colloids. The Crimean clay «kill» ([10]), as well as the American bentonite, are also associated with regions of solonetz occurrence. Ross and Shannon ([11]) have already pointed to the fact of the formation of bentonites in the alkali medium of lakes (most probably in result of the salinization process subsequent to the disappearance of the lakes). Thus, montmorillonite clays are to be found in modern or ancient accumulative solonetz regions, while kaolin clays are connected with alluvial regions. Within the range of the same region, montmorillonite and kaolinite could be formed in result of the lengthy existence of alkali conditions in the first case and an acid medium in the second case.

A study of the mineralogical composition of the colloidal fractions of facies deposits of different geological periods can greatly help in tracing back the history of the evolution of separate regions. Investigating the composition of the minerals of soil colloids helps in the solution of many problems of the genesis and development of new and old soils.

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METEORITICS

DATA ON THE TUNGUS METEORITE AS AVAILABLE TOWARDS 1939

By L. A. KULIK

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The fall of the Tungus meteorite about 7 a. m. on June 30, 1908, was noted by numerous observers who pointed out that the bolide flew in a direction approximately from south to north in a clear sky and calm weather. When the bolide fell in the taiga, a «pillar of fire» mounted up to the sky from the place where it fell and immediately 3 or 4 distinct and powerful blows of incomparable force and timbre were heard accompanied by a roar and rumbling. The blows were heard over an area with a radius of more than 1000 km. They were accompanied also by mechanical effects: the water in the rivers was driven in huge waves, men and beasts were knocked off their feet, many buildings were damaged, fences were torn down, houses shook and objects suspended in them were made to swing. This atmospheric wave was recorded by the barographs (and microbarographs) throughout the world. From the western hemisphere it moved back to the eastern hemisphere, thus having travelled twice over around the globe. The seismographs at that moment recorded an earth wave, the epicentre of which, as it has now been ascertained, was located beyond Tunguska II, where the meteorite fell. The seismograph with a Repsold pendulum in Irkutsk showed an amplitude of about 2 mm and the recording lasted approximately one hour and a half. At dusk, that same day, enormous silver clouds were seen at an altitude of 83 km, which almost turned the night into day throughout the continent, from the basin of the Yenissei to the Atlantic and even the Black Sea.

The expedition undertaken by the writer of these lines in 1921 collected the first data on the fall of the meteorite. We gathered interesting information about windfallen trees and about water gushing from the earth at the place where the meteorite fell. Our expedition also explored the vicinity of the railway siding Filimonovo, near Kansk, where, according to the assumption of the press, the meteorite had fallen. Our exploration, however, disproved this assumption. In 1925 and 1927 all these data were supplemented by the data collected at Tunguska II in 1924 jointly by S. V. Obruchev and I. M. Suslov, as well as by the data collected by A. V. Voznesensky who published the materials of the Irkutsk Observatory of 1908. All this seemed to the author sufficient to undertake a reconnoitring journey to Podkamennaya Tunguska in 1927. Within a hundred kilometres N. W. of the factory Vanavara we established the following facts.

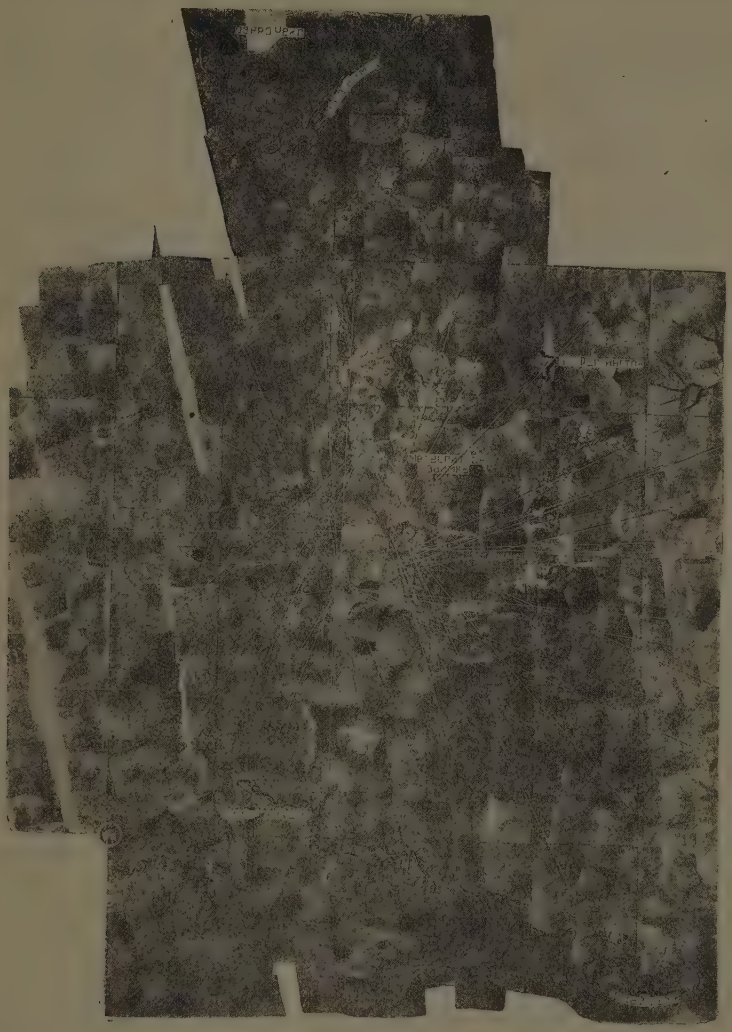
The central part of the site, where the meteorite fell, lies within a distance of 3.5 or 4.5 km S. W. of the Farrington mountain marked later, in 1929, as the astro-radio centre with the co-ordinates: $\varphi = 60°54'58''.98$, and $\lambda = 101°56'59''.70$. This part of the site represents a fairly even platform of about 40 or 50 km² which rises 70 m above the neighbouring valleys of the streams Kimchu and Khushmo, secondary tributaries of the river Podkamennaya Tunguska. The terrace is surrounded by a belt of trap mountains rising to a height of 120 m above it. It is almost a closed ravine with only two outlets north and south in the shape of narrow ancient erosion valleys, of which the southern, evidently of later origin, has a gorge with a waterfall. This locality is practically an undrained hollow. The watershed between the Kimchu and Khushmo basins traverses the locality latitudinally roughly through the centre, lending it the form of a very steep roof. The terrace abounds in hillocky peat marshes, while in its southern half there is a swamp about 5 m deep and covering an area of about 4 or 5 km². On the outskirts of the swamp there are also hillocky peat marsches, of which the most noted ones are two located in the shape of islands in the northern part of the swamp almost on the meridian «Meteorite Field».

The whole region of the river basins Kimchu and Khushmo is covered with windfallen trees lying fanwise in a circle, their tops pointing outwards of the central part of the terrace. The trees are uprooted, broken and all of them without top parts. On the whole this region of windfallen trees coincides with the locality indicated by S. V. Obruchev and I. M. Suslov in their articles on the basis of statements made by Evenks.

The peat marshes of this region are deformed and the whole place bears evidence of an immense catastrophe. The windfallen trees begin at a certain distance from the centre, and between them and the latter there stands a dead forest, its trees devoid of their tops. The whole central region bears traces of a burn. These traces growing gradually weaker are evident even farther from the centre for a distance of 15 or 20 km. More marked are the burns on the side of the trees facing the centre; the tops of the standing trees or what remains of them are in each case burnt, and finally all the trees and bushes show burns at the place where they are broken. The burning of the taiga over an area with the radius of 15 or 20 km explains the fact that the flames were visible in the day time («a pillar of fire») 400km away (at Kirensk). All the circumstances attending the fall, beginning with the 3 or 4 superpowerful blows or explosions, show that we have here an outstanding case in world history, when a huge meteorite broke in parts in the air and the parts driven with theis remaining cosmic velocity buried themselves in the ground causing a number or explosions («blows»).

The expedition of 1927 was merely of a reconnoitring nature, and in 1928 I set out with a new expedition in order to effect a detailed investigation of the windfallen trees, to make surveys and geological studies and film the place. This time the area of the windfall was investigated for a distance of between 25 and 55 km in various directions, and we established the fact that the continuous windfall runs from the centre for distances up to 30 km. Farther on there are sporadic windfalls which are of a heavier nature on the high hills. The higher points were measured with a theodolite, and a scheotic chart of the central area of the fall was drawn up with an indication mathe direction, in whieh the trees fell. The geological study brought to light the presence of permanently frozen ground, the peat marshes showed deformations. The mineral matter taken from the bottom of the swampy lowlands amidst the hillocky peat marshes in the northern half of the central terrace showed the presence of thinly crushed, sharply angular fine matter akin to the rock of the surrounding trap mountains and similar to the «pow-

der» of meteorite craters. The films taken gave an excellent picture of the windfallen trees chiefly in the neighbourhood of the Khushmo stream. The magnet study proved a failure. The same year, i. e. 1928, a base consisting



Field mosaic photoscheme of the fall of the Tungus meteorite the 30.VI.1908. White lines show the direction of windfall and the centers of distribution of explosive air waves.

of three huts («Meteorite Field») was set up at the centre of the terrace on the watershed.

The data obtained by the two expeditions necessitated a third expedition which was effected by the writer in 1929. The object of the expedition was to record the yearly cycle of meteorological observations, to make bore-holes and geobotanical investigations, to take photographs from the air and determine the astro-radio points. The drilling done on the hillocky peat marshes in

the northern part of the terrace facing the «Meteorite Field» showed that in August the permanently frozen ground begins at a depth of half a meter and runs to a depth of 25 m, while farther down there are water bearing layers of pure sand and sand with some admixtures. The lowest depth attained in drilling was 31.5 m. When the water-bearing layer was reached, the water in the drill pipe rose 20 m, thus showing that it was under pressure.

This, as well as the configuration of the locality, explains to us the evidence of eye-witnesses, who maintained that where the meteorite fell it «twisted and turned the ground» and «water gushed from the earth». The evidence about water gushing from the ground is confirmed by traces of a great inundation, when the level of the water in many of the explored places rose above the level of the hillocky peat marshes. In one of the clayey sides of a round depression, 200 m west of the «Meteorite Field», a bluish semi-transparent blistery piece of glass weighing about half a kilogram was discovered, the analysis of which revealed traces of Ni.

The hillocky peat marshes both in and outside the zone of the catastrophe were studied by L. V. Shumilova, the geobotanist, and myself. It has been established that in the peat marshes outside the area of windfallen trees the hillocks predominate over the lowland, the latter having a sphagnous cover while the hillocks reveal a regular lie of parallel layers, the thickness and alternation of which show a relative regularity. The bottom of the lowlands is level, the mineral core of the hillock is slightly convex, the border line dividing it from the peat is sharply defined. In the peat marshes lying within the zone of the continuous windfall, but outside the zone of the burn, the area of the hillocks and of the lowlands are almost equal. The lowlands have a sedge and hypnum cover and a tendency to encroach upon the hillocks, all of which is ascribed to the change in the microclimate as a result of the layers of the permanently frozen ground being laid bare. In the peat marshes within the centre of the windfall and burn the lowlands predominate over the hillocks, not only because of the change in the microclimate but also because of the effect which the inundations that formed the huge Southern Swamp, by disintegrating the hillocks on an area of 4 or 5 km^2, had on the frozen ground. The cover of the lowland swamps varies; hypnum swamps mark lowlands encroaching on hillocks or lowlands without peat hillocks connected by subsoil and surface waters. Through a number of transitional types these swamps change on the hillocks into depressions formed by atmospheric action and having a sphagnous cover with a tendency to encroach on the sides of the depressions. The powerful rise of the waters is also confirmed by the concentric peat bands on the Southern Swamp (a result of the fall of the level of the water), and also by the way the tree trunks were piled up in the valley of the river Churgim at the time the meteorite fell. A section, 38 m long and 4 m deep, made of the hillocky peat marsh 200 m west of the «Meteorite Field», revealed a striking picture of the frozen mass of peat encroaching meridionally on the mineral substratum and forming folds (up to 1 m) with bends and faults and with the component parts of the peat bog, being mixed and ground into «peat dust» involving also parts of the mineral core. In this place the covering layer of vegetation of the hillocky peat marshes has obviously the features of a restitution process. The preliminary data of the meteorological observations have shown a minimum temperature of minus 55.6° C (12. XII. 1929), a maximum layer of snow, averaged for the whole terrace of 63 cm, and noted that on January 12,1930, the upper layers of the soil froze right down to the permanently frozen ground. Astronomer S. I. Belykh determined 3 astro-radio points: at the factory Vanavara and on the mountains Shakrome and Farrington. The expedition did not succeed in taking photographs from the air in 1930. It was not before 1938

that such photographs were taken by the Chief Administration of the Northern Sea Route. It aimed, while showing the position of the windfallen forest, to determine the loci of the fall of separate meteorite masses, and to obtain objective evidence as to the fall.

The photographing was done between June 26 and July 18, 1938, when the taiga already bore a quite developed leaf-cover, which certainly reduced the visibility of the windfallen trees on the photographs which are on a scale of 1 : 4700 with a geodetic foundation of the V class. The photocuts and then the mosaic field photoscheme showing the centripetal direction of the windfallen trees enabled to establish the starting points of the air waves. Naturally enough, this centre coincided with the centre determined by me in 1928 by direct theodolitic measurement of the windfallen trees. As to the particular points of the explosion, the field photoscheme so far indicated between 2 and 4 of them, the principal points lying S. W. and N. E. of the northern cape of the Southern Swamp (figure). When the photomaterial is studied and after a photochart more perfect than the field photoscheme is made, will these indications become more accurate. Besides the field deciphering of the photographs taken from the air and the supplementary geodetic work necessitated by the fixing of the points (areas), where the separate parts of the meteorite exploded, physical methods of investigation should be applied in order to confirm the correctness of the conclusions drawn from the aerophotographs and also in order to specify the location of the buried parts of the meteorite; only then will it be possible to raise the question of extracting these buried parts. Due note should be taken of the following.

Permanently frozen ground extends all over the locality. The principal points where the pieces of the meteorite fell lie in a place which has since become swampy. Usually craters formed by the fall of meteorites under such conditions (water gushing from the ground and the place being turned into a swamp) become levelled. The average dimensions of such craters may range from 50 and 100 m. The meteorite masses forming each crater should weigh thousands of tons at the moment they touch the earth, but taking into consideration the fact that, while burying themselves in the ground they become crushed and split, the biggest parts of meteorite should not weigh more than a hundred or two hundred tons. Taking into consideration the general picture of the fall and the resistance power of the frozen ground, the depth to which the particular meteorite parts penetrated the ground should vary between 25 and 50 m. The substance of the meteorite is supposed to be nickelous iron (iron meteorite) on the basis of the following facts: history does not know stony meteorites weighing more than half a ton; only iron meteorites—we speak of those which have been studied—weight scores of tons (14, 27, 31, 60 tons); all known meteorite craters have been formed only by iron meteorites; immediately after the fall of the meteorite, Evenks found pieces of white glistening metal, the size of a fist, on the charred soil in the centre of the windfall. According to their description and the given conditions, these pieces could be nothing else but pieces of an iron meteorite; in the bottom mud of the Southern Swamp, near the eastern centre of the fall, the author detected under the microscope occasional silver white globules of malleable nickelous iron which is associated with molten groups and bunches of round grains of quartz.

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GENETICS

MANIFESTATION AND EXPRESSION OF THE EFFECT OF GENES POSSESSING NON-AUTONOMOUS ACTION

By M. E. NEUHAUS

(Communicated by I. I. Schmalhausen, Member of the Academy, 1.II. 1939)

I. MANIFESTATION OF THE GENE *cn* IN *DROSOPHILA MELANOGASTER*

1. The term «gene possessing non-autonomous action» is applied to cases when primary or secondary products of a given gene are able to produce their effect not only in those cells where they were formed, but in other cells also. The term «gene possessing autonomous action» should be, accordingly, applied to such a gene whose products exert their effect only upon the cell, in which they were formed. It is interesting to note that certain genes possessing pleiotropic effect as shown by our studies (unpublished) appear to be autonomous with regard to some characters, while they are non-autonomous with regard to other characters.

2. It is known([1,3]) that when larvae $w^a v$ or $w^e cn$ are given food with admixture of pulp of pupae of the wild line, the eye colour of the flies is shifted in the direction of w^a or w^e. Proceeding from this fact, one could admit that in case normal larvae and larvae possessing the genotype $w^e cn$ develop in a common vial, the eye colour of the $w^e cn$ flies should change in the direction of w^e. In favour of this assumption speaks, first, the fact that many of the larvae die prior to pupation and may, therefore, serve as food to others. Secondly, it is known([2]) that the substance $\dot{c}n$ is present in Malpighian vessels and, since the latter are excretory organs, the possibility is not excluded that normal larvae give off the substance $\dot{c}n$.

3. A number of experiments of the following sort were conducted. Larvae of the line $w^e cn$ were placed into a vial, where larvae of the wild line Florida were developing. The eye colour of the flies of the genotype $w^e cn$ which hatched in these vials was strongly changed in the direction of w^e. It was noted that during the first days of hatching the eye colour was not very much changed. During the following days, however, the eye colour was identical with w^e. It was observed, besides, that when the larvae developed at a temperature 18—22° the eyes were coloured less intensely, than when the larvae developed at a temperature 25—28°.

4. Starting with the above facts, one should expect modification of the theoretical ratio 3 w^e: 1 $w^e cn$, typical of the cross between heterozygotes for the gene cinnabar. In order to check this point, the following experiment was made. Flies $w^e cn\ b$ (the eye colour in such flies is white with a barely perceptible shade of yellow) were crossed with flies w^e (in the females eyes are pink,while in males they are yellowish pink). In F_1 all flies possessed w^e eye colour, as should be expected. To obtain the F_2, flies of the F_1 were intercrossed. Equal numbers of females and males were placed into standart vials containing the usual food.

Results of the Crossing ♀♀ $\frac{w^e}{w^e}\ \frac{b\ cn}{+\ +}$ × ♂♂ $\frac{w^e}{b}\ \frac{b\ cn}{+\ +}$

Series number	w^e	Eye colour intermediate between w^e and $w^e cn$	$w^e cn$	$\frac{+}{b}$	b	w^e	Eye colour intermediete between w^e and $w^e cn$	$w^e cn$	$\frac{+}{b}$	b
	1 heterozygous ♀					2 heterozygous ♀♀				
I. Obtained	336	25	93	299*	109	271	24	56	261	87
Expected	340.5	0	113.5	306	102	263	0	87.75	261	87
II. Obtained	390	3	140	374	150	301	31	65	250	86
Expected	399.75	0	133.25	393	131	297.75	0	99.25	252	84
V. Obtained	88	0	31	89	32	130	1	55	124	56
Expected	89.25	0	29.75	89.25	29.75	139.5	0	46.5	135	45
Total: obtained	814	28	264	762	291	702	56	176	635	229
expected	829.5	0	276.5	782.25	260.75	700.5	0	233.5	648	216
	4 heterozygous ♀♀					8 heterozygous ♀♀				
I. Obtained	581	73	62	532	184	186	24	18	169	47
Expected	537	0	179	537	179	171	0	57	162	57
II. Obtained	609	87	87	570	183	271	45	22	254	83
Expected	587.25	0	195.75	564.75	188.25	253.5	0	84.5	252.75	84.25
III. Obtained	194	26	12	166	66	160	20	5	144	41
Expected	174	0	58	174	58	132.75	0	44.25	132.75	44.25
IV. Obtained	130	15	38	132	50	207	35	40	198	84
Expected	137.25	0	45.75	136.5	45.5	211.5	0	70.5	377.25	70.5
V. Obtained	150	36	43	149	59	142	47	39	140	52
Expected	171.75	0	57.25	156	52	171	0	57	144	48
VI. Obtained	—	—	—	—	—	124	17	15	106	48
Expected	—	—	—	—	—	117	0	39	115.5	38.5
VII. Obtained	—	—	—	—	—	171	28	26	169	56
Expected	—	—	—	—	—	168.75	0	56.25	168.75	56.25
Total: obtained	1664	237	242	1549	542	1261	216	165	1180	411
expected	1605.25	0	535.75	1568.25	522.75	1231.5	0	410.5	1193.25	397.75
	16 heterozygous ♀♀					32 heterozygous ♀♀				
I. Obtained	250	34	23	193	69	100	12	7	96	33
Expected	230.25	0	76.75	196.5	65.5	89.25	0	29.75	89.25	29.75
II. Obtained	274	31	25	244	86	189	15	22	169	55
Expected	247.5	0	82.5	247.5	82.5	169.5	0	56.5	168	56
III. Obtained	236	18	22	207	67	148	12	7	124	41
Expected	207	0	69	205.5	68.5	125.25	0	41.75	123.75	41.25
IV. Obtained	381	55	67	348	155	150	26	24	140	60
Expected	377.25	0	125.75	377.25	125.75	150	0	50	150	50
V. Obtained	121	13	44	121	58	104	18	33	102	53
Expected	133.5	0	44.5	134.25	44.75	116.25	0	38.75	116.25	38.75
VI. Obtained	258	27	23	236	72	98	14	4	82	32
Expected	231	0	77	231	77	87	0	29	85.5	28.5
Total: obtained	1520	178	204	1349	507	789	97	97	713	264
expected	1426.5	0	475.5	1392	464	737.5	0	245.75	732.75	244.25
	64 heterozygous ♀♀									
VII. Obtained	221	39	30	205	85					
Expected	217.5	0	72.5	217.5	72.5					
Total: obtained	221	39	30	205	85					
expected	217.5	0	72.5	217.5	72.5					

* The eye colour was determined immediately upon hatching, the body colour—only 24 hours later; this probably accounts for the fact that the total number of flies in the first count was greater, than in the second one.

The eye colour in F_2 flies was thoroughly recorded. All the data obtained are summarized in the table. One should note after inspection of this table, first, that, aside from w^e and w^ecn flies such appeared possessing an eye colour which was intermediate between w^e and w^ecn. In this group individuals were included which possessed eye colour darker than in w^ecn and lighter than in w^e. The number of such individuals was not great in vials where less than 4 mothers were present, while their number was relatively great in cultures where there were more than 4 mothers. Not only the expression of the action of the gene cinnabar had changed, but in many cases the gene *cn* was not manifested at all. This may be seen from the fact that much more w^e flies occurred in the cultures where more than 4 mothers were present. It might be inferred that the viability of the w^ecn flies was lowered as compared to the w^e flies. However, this was not the case, as the flies homozygous for the gene black (which is located at a distance of 10 morganids from the gene cinnabar) were no less numerous than expected. On the contrary, there occurred more black flies than expected. Apparently, flies of the genotype w^ecnb are more viable than w^e. Our supposition that manifestation and expression of the gene cinnabar should change when flies heterozygous for *cn* are crossed with each other is thus justified. Curiously enough, although the expression of the gene cinnabar has changed in series V, VI and VII, suppression of the manifestation of the gene cinnabar was not always noticeable. Perhaps, it should be explained by the fact that the cultures belonging to these series were accidentally developing at 18—20°. This point is being verified at present.

5. In flies homozygous for the genes w^ecn variation in eye colour was observed, although not a very great one. Individuals with white eyes were found, as well as those possessing eyes of various yellowish shades (only young flies were compared). The proportion of such individuals was especially high in mass cultures. This fact may be explained on the assumption that the allelomorph of cinnabar which was involved in our studies is hypomorphic. The larvae w^ecn contain, therefore, a small amount of the substance $\check{c}n$.

6. Genes with non-autonomous effect are relatively rarely met with in *Drosophila melanogaster*. Out of the 25 genes influencing the eye colour([4]) only two proved to possess non-autonomous effect. Among mutations, very often produced by geneticists in *Drosophila melanogaster*, the so-called semi-lethals are met with. Among these there are apparently true lethals; however, they are classified with semi-lethals as their normal allelomorphs possess non-autonomous action. Since the larvae homozygous for these lethals develop in a common medium with normal ones, it is possible that the latter are sometimes consumed by those homozygous for the lethal gene, owing to which fact the latter survive.

It is possible also that imperfect expression or manifestation of certain mutations in *Drosophila* should be explained by the fact that these mutations are hypomorphic and the products of these genes possess a non-autonomous effect.

Conclusions. 1. When normal larvae are reared together with those of the consitution w^ecn in a common vial, the eye colour in w^ecn is changed in the direction w^e.

2. Manifestation and expression of the action of the mutant gene *cn* is very much changed in animals derived from parent of the genotype w^e *cn*.

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PLANT PHYSIOLOGY

DISTRIBUTION OF BIOS IN LEAF OF PUERARIA AND VINE

By J. V. RAKITIN and L. M. JARKOVAJA

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It is a well established fact now(5) that the formation of bios proceeds in the chlorophyll-bearing cells under the influence of the strongly refrangible part of the solar spectrum. We know, moreover (5), that bios can be adsorbed by protein and that its disappearance from or else its storage in the solution is determined by the condition of the protein. Comparative estimation of bios in various parts of plants has shown the leaves to be the first in its storing(2,3).

Being continuously high in bios, the leaves change in its content as they develop, and the curve of variation has, according to Dagys (4), two maxima. The first corresponds to the period when leaves are growing, the second is bound up with the processes of their dying away.

The object of the present work was to ascertain the amount of bios in the leaves when the latter had either completed growth or were ready to fall in autumn. The leaves, supplied for the experiment in dry condition from Batumi, belonged to vine Isabella and to *Pueraria Thunbergiana*.

Pueraria, a leguminous climber, has been brought to the Black Sea coast from Japan. The plant has become perfectly inured to its new station and has spread over many littoral mountain slopes covering them with the thick green carpet of its foliage. For the test the leaves both of vine and pueraria were cut off with small portions of stem (Fig. 1 and 2).

The first experiment was run with various parts of full-grown leaves from pueraria. The content of bios was estimated in: 1) the leaf blades, 2) the short swollen petioles connecting the leaf lobes with the main petiole, 3) the main petioles, 4) the swollen bases of the main petioles, 5) the stem portions cut off together with the leaves. To prepare the extracts 1 g samples of air dry material were taken and each of them was boiled in 20 cm^3 water for 3 minutes. The extracts thus obtained were decanted into graduated flasks and made to 50 cm^3 by adding water. Bios was estimated in them after the yeast method of Verner and Kling(1): 2 cm^3 of extract were introduced into Hayduk's nutrient medium inoculated with yeast. The increase of yeast cells was estimated quantitatively after 48 hours (by means of Hellige's counting chamber).

The results of the experiment are referred to in Table 1 (average number of yeast cells from 10 counts in the field of the counting net of Hellige's chamber—objective 14, Hellige eyeglass).

Table 1

	Average number of yeast cells
Stems	200
Swelling at the base of main petioles	575
Main petioles	375
Swellings of the petioles connecting the leaf lobes with the main petioles	550
Leaf blades	850

As seen from the table, the maximum content of bios has been stated in the leaf blades. The second place as to the content of bios is occupied by the swellings at the base of main petioles and the swellings of the petioles connecting the latter with the leaf lobes. As third come the main petioles, and the stems are the last in this series.

The second experiment was run with various parts of vine leaves. Bios was estimated in: 1) the leaf blades, 2) the swelling at the base of main petioles, 3) the upper part of the petioles, 4) the petioles, 5) the stems. The material for the test was taken twice from the same plant: 1) in summer (full grown green leaves); 2) in autumn, shortly before falling (with first signs of yellowing). The extracts were prepared from air dry material—0.3 g for each sample—in the same way as described above. 1 cm^3 was the volume of each of the extract portions introduced into Hayduk's medium inoculated with yeast. The amount of yeast increased in 48 hours was counted. Table 2 presents the results of the experiment.

Table 2

	Number of yeast cells		
	Summer samples	Autumn samples	Note
Stems	15	20	Method of counting as in Table 1
Swellings at the base of the petioles	40	120	
Petioles	25	56	
Upper parts of the petioles (1 cm)	35	95	
Leaf blades	60	160	

The table shows that the distribution of bios in green leaves of vine is governed by the same regularity as has been established with regard to pueraria.

Our attention is engaged by the fact that, besides the blade, those parts of the leaf are relatively high in free bios which perform the function of growth movement due to the geo- and phototropic effect. Such are the lower swollen parts of the petioles and the swellings of the petiole connecting the leaf lobes with the main petiole in pueraria and the lower and upper parts of petioles in vine.

Figs. 1 and 2 afford a representative illustration of the presence in the leaves of vine and pueraria of portions preforming the function of growth.

From Table 2 it may be seen that towards the period of leaf fall the leaves increase sharply in free bios. And it is worthy of note that while in various portions of the leaf the content of bios amounts by that time to 2—3 times of its former value, its increase in the stem is only 33%. Since the autumn increase in the bios content of leaves is associated with their dying away,

we have to suppose that this increase is due to the enhanced break-down of the protein and the transition of the combined bios into free state rather than to intensified formation of new bios. If so, we can get an idea of the depth of the break-down processes from the relative content of bios.

Interpreted in terms of this dependence, the data of Table 2 furnish evidence that the processes of break-down in general grow more intense towards the time of the autumn fall of the leaf. Therefore they attain their maximum strength in the bases and are most weakly expressed in the stems. Another valuable evidence afforded by the dependence recorded is that the shedding of leaves is associated with a sharp increase of disintegration within the leaf itself. From the order in which vine and pueraria shed the separate parts of their leaf we are entitled to say that the processes of decay manifest themselves in the blade with special strength, and it is only later that they

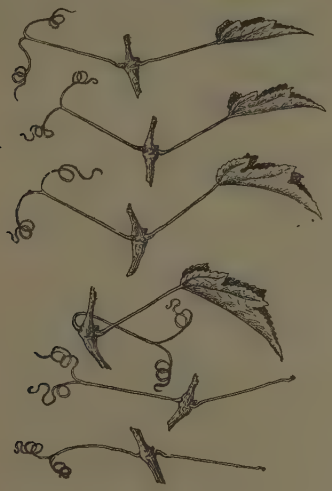

Fig. 1. Leaves of vine.

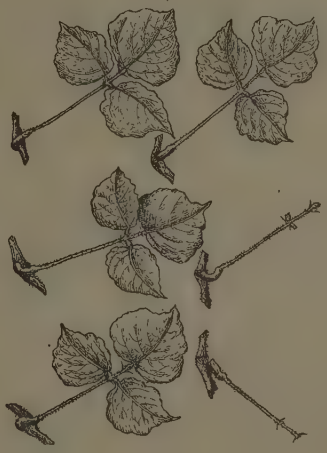

Fig. 2. Leaves of pueraria.

become apparent in the petiole. From Fig.1 we see that in vine the leaf blade is the first to fall. The fall of the blade is determined by the formation of a separating layer at the place it is fixed to the petiole. Later on falls the petiole, after such layer has formed at its base.

Similar picture has been observed with pueraria (Fig. 2). First fall the blades of the ternate leaf which are later followed by the petioles. The reason to account for this order is that in both vine and pueraria the formation of the separating layer at the places of fixation of each of the leaflets in the ternate leaf is ahead of the development of such layer at the base of the main petiole.

In other words, the formation of separating layer is effected with greater speed in those parts of the petiole which first meet with the mass of bios, set free on disintegration of proteins in the leaf blade.

It is known from vegetable anatomy that the separating layer is usually formed out of the meristem which arises from differentiated tissues. Consequently, the creation of meristem, which then turns into separating layer, is ordinarily a result of cell division. It is therefore obvious that during the formation of meristem some stimulators of the cell division should be acting. The order in which vine and pueraria shed the separate parts of their leaf,

as well as the changes in the content of bios related to this phenomenon, lead us to the conclusion that the most probable stimulators of the cell division are substances, whose formation is bound up with the intensification of break-down processes in various tissues of the leaf and first of all in the leaf blade tissue. Very likely the bios set free on disintegration plays the rôle of one of those substances which are responsible for the outset of the cell division.

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PALEOZOOLOGY

ON THE STRUCTURE OF EXTREMITIES OF *PARATAXIDEA CRASSA* ZDANSKY

CONTRIBUTION TO THE PHYLOGENY OF *MELINAE*

By J. A. ORLOV

(Communicated by A. A. Borissiak, Member of the Academy, 4.II.1939)

Among the material on *Carnivora* obtained by the Paleontological Institute of the Academy of Sciences of the USSR, as a result of excavations made in the locality of Hipparion fauna at Pavlodar (3) on the Irtysh river, remains of a badger deserve great interest. Extremities of fossil *Melinae* from Asia have not been described until now: neogene badgers of the genus *Parataxidea* were assumed to have been adapted to aquatic life(4). Moreover, this genus presents special interest because of its being one of a few *Carnivora* of the Hipparion fauna common to North America and Asia(4).

L o c a l i t y: Neogene deposits with Hipparion fauna on the right bank of the Irtysh river, at Pavlodar, by «Gussiny Perelyot».

H o r i z o n: argillaceous sand dividing ochreous micaceous sands and clays overlying the latter [bed № 2 of the section, (3)].

M a t e r i a l: collection of the Paleontological Institute of the Academy of Sciences of the USSR № 2346 (1929). Bones of extremities belonging in the main to the same old individual as the skull (№ 2346/166) and the mandible (№ 2346/168).

F o r e E x t r e m i t y

H u m e r u s, while showing general resemblance to the humerus in *Meles*, is distinguished by: 1) a less broad fossa olecrani; 2) less developed internal condyle in the median direction; 3) smaller width of the entire distal end; 4) lesser development of the crest separating the pulley (the trochlea) at the level of the posterior side of the arm. It should be admitted that the arm of *Parataxidea* bears characters of lesser specialization (a smaller degree of adaptation for burrowing), than the arm in *Meles*.

R a d i u s. Proximal and distal ends less massive than in *Meles*, the entire bone more slender and seems to be less specialized than in *Meles*.

U l n a. Olecranon shorter and like the entire proximal end of the bone less strong than in *Meles*; the length of the bone from incisura semilunaris to the distal end greater than in *Meles*. Thus, m. triceps, the extensor of the

forearm, was in *Parataxidea* attached to a shorter lever than in *Meles*, and the extremity, if only on account of this peculiarity, was less adapted to burrowing than in *Meles*.

Scapholunare, with region corresponding to radiale, somewhat less massive than in *Meles*:

Triquetrum in *Parataxidea* and *Meles* quite similar in size and form of articular surfaces.

Magnum articulates with unciforme by a smaller facet, while the transverse diameter of the bone is somewhat greater, both absolutely and relatively, than in *Meles*.

Unciforme — of the same proportions, though relatively and absolutely smaller than in *Meles*.

Pisiforme. Base of the bone like that in *Meles*; shaft and distal end less massive than in *Meles*, probably due to a lesser development of m. flexor carpi ulnaris, and to a certain extent of m. abducens digiti V.

Metacarpalia II, III and IV very much like the corresponding bones in *Meles*. Matacarpalia I and V, especially Mc V, slimmer and shorter than in *Meles*, the general form of the shaft and epiphyses of the wide and flat Mc V in *Parataxidea* identical with those in *Meles*, the bone already bearing all the characters of specialization of the *Meles* type. Mc IV is also relatively less massive than in *Meles*. Distal ends of metacarpal bones exhibit a more pointed and distinctly modelled keel for the articulation with phalanges of the proximal row than in *Meles*.

Hind Extremity

Femur slenderer than in *Meles*; head, tr. major, tr. minor and crista intertrochanterica smaller, but fossa trochanterica larger than in *Meles*. On the other hand, the summit of tr. major in *Parataxidea* lies on a line with the proximal point of capitulum femoris; in *Meles* the great trochanter is reduced in the proximal direction. Such expansion of tr. major in the proximal direction due to the strong development of m. glutaeus medius and m. obturator internus occurs in mammals with an entirely different adaptation of extremities (seals, artiodactyls); the significance of this particular character in *Parataxidea* remains unclear for the present. The distal ends of the thigh in *Parataxidea* and *Meles* are similar in size and proportions.

Tibia (distal end)—quite similar to that in *Meles*.

Fibula. The shaft of the bone is flattened as compared to the one in *Meles*; the outer surface of the proximal end bears a large pit for the attachment of the fleshy head of m. peronaeus longus.

Astragalus—of the same size and shape as in *Meles*.

Calcaneum—somewhat shorter than in *Meles*, mainly on account of a slight reduction of the shaft of the bone.

Naviculare — position relatively and absolutely lower (possessing a smaller proximo-distal diameter) than in *Meles*.

Cuneiforme III (Ectocuneiforme) differs from that in *Meles* by: 1) a concave facet for naviculare; 2) less slanting of this facet in the medial direction; 3) weak development of unciform process on the plantar side for m. peronaeus longus, and of the whole plantar portion of the bone.

Metatarsalia I and II missing. Mt III differs from that in *Meles* by a straighter shaft, greater width and smaller sagittal diameter of the proximal end. Mt IV, on the contrary, possesses a proximal end with a greater sagittal diameter, owing to a strong expansion of the epiphysis

in the plantar direction. Mt V is distinguished by articular facet for Mt IV being strongly developed and more convex forwards. Evidently, the Mt V was more movable in *Parataxidae* than in *Meles*.

Phalanges of the distal row—of shape peculiar to a badger with digging extremities; differ from the corresponding phalanges in *Meles* by a greater lateral compression at the dorsal side of the base.

Conclusion

Parataxidea is a typical badger, as far as the specialization of its extremities is concerned. The specialization in badgers is manifestly higher in the fore extremities owing to their burrowing functions, whereas the hind extremities of *Parataxidea* and *Meles*, being less specialized, bear more likeness to one another in the two genera than the fore extremities.

The fore extremity in *Parataxidea* is characterized by a lesser degree of specialization (adaptation to burrowing) than in *Meles* (see description of humerus, ulna, metacarpalia, pisiforme). Contrary to Zdanky's ([4]) assumption based on the study of the skull alone, there is no evidence whatsoever speaking for the adaptation of *Parataxidea* to aquatic mode of life.

The specialization of teeth ([4]) excludes *Parataxidea* from the direct ancestral line of the contemporary *Taxidea* of North America, regardless of the primitiveness in other respects as compared to *Taxidea*. *Parataxidea* is rather a lateral branch of the trunk wherefrom the modern *Taxidea* has arisen. The typically badger-like structure of the extremities in *Parataxidea* speaks against Pocock's (1920) assumption as to the purely convergent character of the resemblance between *Meles* and *Taxidea*. The occurrence of not less than three genera of *Melinae* (*Trochictis* v. Meyer, *Trochotherium* Fraas, *Trocharion* Majoris) during the Miocene of Western Europe, two genera in the Pontian of Europe (*Promeles*, *Promephitis*), Asia (*Parataxidea*, *Meles*) and North America (*Taxidea*) points to a relatively early division of the stock of *Melinae* into various branches, most of them extinct by now, which had passed into the Upper Miocene—Lower Pliocene with a distinct badger type specialization.

If Zdansky's assumption regarding the close relation of *Parataxidea* to the European *Trochictis* from the Miocene of Sansan ([2]) were confirmed, this would argue for the Upper Miocene age of *Parataxidea*, as well as Hipparion fauna from China and Siberia. The study of the Hipparion fauna from Siwalik Hills, and especially comparison of Hipparion from the Old World with their most probable direct ancestors from North America has led, however, Colbert ([1]) to the firm belief that the time of appearance of Hipparion in Asia most probably goes back as far as the Pontian or Upper Sarmatian, at the earliest, and that badgers of the genus *Parataxidea* must have arisen at no earlier time than that.

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PALEOZOOLOGY

ON DENTITION AND EXTREMITIES OF *ICTITHERIUM HIPPARIONUM* GERVAIS

CONTRIBUTION TO THE PHYLOGENY OF *ICTITHERIINAE*

By J. A. ORLOV

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Notwithstanding the wide paleogeographical range of distribution of *Ictitheriinae* and studies carried on since many years past, some features of their organization have remained somewhat obscure up till now. Nor is the taxonomic position of *Ictitheriinae* quite clear, as they are being referred sometimes to *Viverridae* and sometimes to *Hyaenidae*. The non too abundant material available in the collections of the Paleontological Institute of the Academy of Sciences of the USSR from the locality of Hipparion fauna at Pavlodar still throws some light on the aforesaid questions. Besides, it is the first and for the time being the only material on *Ictitherium* for all the Asiatic part of the USSR.

L o c a l i t y: right bank of the Irtysh river at Pavlodar near «Gussiny Perelyot» (Western Siberia).

H o r i z o n: crossbedded micaceous ochreous sands and argillaceous sand bordering on ochreous sands overlain by marly clays [(1), beds 1—2, Fig. 2—3].

M a t e r i a l: collections of the Paleontological Institute of the Academy of Sciences of the USSR, № 2346 (1929) and 2413 (1930).

T e e t h

The teeth of *I. hipparionum*, though less specialized than in large hyaenas, still are bearing unquestionable marks of the same adaptation. The enamel of the upper and lower teeth is covered on the surface with diminutive wrinkling, particularly distinct on the large cusps of the anterior and posterior molars. The conical main cusps of the upper and lower premolars have folds of the enamel diverging from the summit towards the base of the crown, slightly winding and gradually branching in a dichotomous manner with anastamoses of the adjoining branches. The enamel folds extend from the main cusps to the vertical styles on the anterior and posterior sides of the cusps, which sometimes gives the impression of a very fine crenelation of the edge, bearing remote resemblance mainly to crenelated canines in *Machairodontinae*. However, the latter should not be confused with the crenelation of the teeth edges in *Ictitherium* and *Hyaena*: in

Machairodontinae the denticles are large, regular, the rest of the enamel surface being perfectly smooth. The folding of enamel in *C. eximia* and in the modern (as well as apparently in all the fossil) hyaenas is of exactly the same character as in *Ictitherium* (it being most clearly pronounced in teeth of young animals; smoothing out with age through grinding). Evidently, this is a case of special adaptation of teeth, peculiar to the entire family of hyaenas and developing convergently in a number of other animals, both carnivorous and herbivorous. Such minute folds, typical as they are for permanent teeth of the family *Hyaenidae*, are wanting in the teeth of *Viverridae* and in milk teeth of hyaenas, far more viverrid in form than permanent ones. It is noteworthy that the modern African «Aard Wolf» *Proteles*, very hyaena-like in outward aspect, has permanent teeth which have lost not only their specialization in the hyaenid fashion, but also to a considerable extent the appearance peculiar to teeth of *Fissipedia* in general. However, the milk teeth of *Proteles* are distincty hyaenid in shape which, according to Brehm (Tierleben), fully confirms the evidence in favour of placing *Proteles* among the hyaenas, regardless of the opinion of some authors ([2]) who consider the Aard Wolf a member of the family *Viverridae*. Whatever may be the case, the ontogeny of the molars in *Ictitheriinae* and *Hyaenidae* brings forth the development of viverrid characters expressed in: 1) the shape of the teeth, and 2) the absence of sculpture on the enamel surface in the milk dentition, peculiar but to permanent teeth in hyaenas, and developed no doubt in hyaenas after the splitting of the common ancestral stock of ancient Viverroids into *Viverridae* and *Hyaenidae*. At the same time the ontogenesis of the teeth in hyaenas furnishes new proof of the close relationship of these two branches of the *Aeluroidea*.

The mandibles in the Siberian specimens of *I. hipparionum* are slender as compared with the material from the Maeotis of the north-western coast of the Black Sea [Grossoulovo ([4]); Emetovka; collections of N. O. Burchak-Abramovitch in the materials of the Academy of Sciences of the Ukrainian SSR]. These distinctions, manifest irrespective of the age of the individual, size and shape of the teeth being completely identical, should be explained by the considerable variation of *I. hipparionum*, which in connexion with the wide range of the Pikermi fauna, extending from Western Europe to China, is probably not less than, say, in the modern *C. lupus*, *Hyaena* and many other carnivores of to-day.

Fore Extremity

H u m e r u s (№ 2413/29) is longer than in the striped hyaena and but somewhat shorter than in the spotted one. Tuberositas deltoidea is less strong, the total width of the distal end and its articular surfaces relatively less than in *Crocuta* and *Hyaena*. Foramen supratrochleare large, the entire shoulder of the same shape as in *Crocuta* but more slender. There are only rudiments of the osseous bridge closing the fentepicon dyloideum. Like in all hyaenas, the surface for the insertion of m. extensor digit. communis, m. ext. dig. V, mm. extensores carpi longus et brevis, m. pronator teres, m. flexor digit. profundus and flexor carpi radialis—relatively smaller than in *Viverridae*.

R a d i u s (2413/31 of the same individual) straighter, slimmer and relatively longer than in contemporary hyaenas, the structure and shape on the whole such as in *C. eximia*. The position of the tubercle for m. biceps is more proximal than in *Crocuta* and *Hyaena*, consequently m. biceps is relatively weaker than in the latter. The groove for tendo m. extens. Mc I—as deep as in *Viverra*.

Ulna (2413/32 of the same specimen) very much like that in *Crocuta*, save for its being straighter, more slender, and fitted in the lower half with longitudinal crests.

Pisiforme rather resembling the slender pisiforme in *H. striata*. Length reduced against that in *Crocuta* and *Hyaena*, tells of relatively less powerful m. flexor carpi ulnaris and m. abducens digiti V in *I. hipparionum*.

Scapholunare of smaller size, though of the same shape as in *Crocuta* and *Hyaena*. Some details testify to a possibly greater flexibility of the fore limbs than in *Crocuta* and *Hyaena*.

Metacarpalia. Mc II bears tuberosity for ligament Mc I (suggestive of rudimentary 1st digit). Mc IV, especially Mc V—essentially similar to the corresponding metacarpal bones in *Crocuta*.

Astragalus—more like that in *Hyaena striata*, but distinguished by a relatively greater proximo-distal diameter of the shaft and a longer neck (features pointing to primitiveness).

Calcaneum—bearing greatest resemblance to calcaneum in *C. eximia*.

Scaphoideum—most similar to that in *C. eximia*.

Metatarsalia, while exhibiting a general likeness to metatarsalia in *Crocuta* and *Hyaena*, are characterized by greater slenderness and consiperable length of bones. Thus, the absolute length of Mt IV and Mt V (№ 2413/38 and 2413/37, one individual) exceeds that in adult specimens of *C. crocuta* (from collections of the Zoological Institute of the Academy of Sciences of the USSR).

Conclusion

I. hipparionum possesses a dentition typical of *Hyaenidae*; the ontogeny of molars in *Hyaenidae* clearly bears out the derivation of *Hyaena* from primitive Viverroid stock and the close affinity between *Hyaenidae* and *Viverridae*. The likeness of extremities of *I. hipparionum* and large hyaenas, casually mentioned by Schlosser ([3]), is traceable through a number of details (especially in *C. eximia*), and cannot be accounted for, just as the similarity in the structure of teeth, by convergence always manifested in more superficial features. The foregoing data as well as finds of *Ictitherium* in the Baluchitherium beds in Mongolia ([5]) lead to the suggestion that ancestors of all hyaenas should be traced in the Old World. The subdivision of the common stock of primitive «viverroids» when still inclined to *Miacidae* must have occurred long before the Upper Oligocene, since in the Baluchitherium beds of Mongolia (Upper Oligocene) are already encountered big, highly specialized *Hyaena* and *Ictitherium*.

The occurrence in relatively young deposits of Eurasia (Upper Sarmatian-Pontian) of a number of more primitive and diminutive species of *Ictitherium*, morphologically transitional between *Viverra* and *Hyaena*, can be explained not merely by the «survival» of primitive forms, in the *Okapia* fashion, but also by repeated radiation of *Viverridae* and a later branching off from the primary *Viverridae* stock of small *Ictitheriinae* of the *I.* D'Orbignyi type, standing nearer to *Viverridae* than to *Hyaenidae*.

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PALEOZOOLOGY

ON THE STRUCTURE OF EXTREMITIES OF *CROCUTA EXIMIA* ROTH ET WAGNER

CONTRIBUTION TO THE PHYLOGENY OF *HYAENAE*

By J. A. ORLOV

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The overwhelming majority of works dealing with Tertiary *Hyaenae* contain information regarding the structure of their skull and teeth, treating quite superficially of the postcranial skeleton; whereas the study of the latter, and of extremities in the first place, is quite indispensable for a more thorough understanding of the constitution of the animal, its mode of life, the reconstruction of its appearance, as well as for the most accurate reestablishment of phylogenetic correlations. Collections of the Paleontological Institute of the Academy of Sciences of the USSR from the locality of Hipparion fauna, at Pavlodar, contain non too large material on the extremities of *Crocuta eximia* Roth et Wagner, though incomplete and fragmentary, but in view of the aforesaid consideration of an unquestionable interest.

L o c a l i t y: Neogene deposits with fauna Pikermi on the right bank of Irtysh at Pavlodar, near «Gussiny Perelyot».

H o r i z o n: crossbedded ochreous micaceous sands and argillaceous sand [(^{3},^{4}), beds № 1 and 2 of section 1].

M a t e r i a l: collection of the Paleontological Institute № 2346 (1929) and № 2413 (1930).

F o r e E x t r e m i t y

R a d i u s distinguished from that in modern and cave hyaenas by: 1) greater length; 2) slenderness and straightness; 3) appreciably smaller development of tuberosities for the attachment of the interosseous ligament uniting the radius to the ulna; 4) by a deeper and better modelled facet on the distal end for reception of the ulna, partly also of circumferentia articularis for the proximal end of ulna; 5) relatively more proximal position of the tubercle for the head of the m. biceps (exactly the same distance from the proximal end as in the spotted hyaena), the forearm being longer than in *C. crocuta*; 6) less expanded part for the attachment of m. supinator longus (on the distal third); 7) very deep and distinctly modelled groove for m. extensor ossis metacarpi pollicis; 8) large size surface for the origin of m. flexor digitorum profundus on the ulnar side of the bone on account of a less marked roughness for ligamentum interosseum. Thus,

with m. biceps relatively weaker, the forearm in *C. eximia* could have possessed more freedom of pronation and supination, and possibly, a comparatively movable, though strongly reduced I digit. Length: in *C. eximia* (№ 2413/30)—232 mm, in *C. crocuta* (Zool. Inst. Acad. of Sc., USSR, № 10813)—209 mm.

Scapholunare—proximo-distal and antero-posterior diameters greater than those in spotted hyaena, obviously because of greater length of the fore extremity.

Unciforme somewhat smaller, possessing a more laterally compressed pr. uncinatus, weaker facet for pisiforme. The latter circumstance, possibly, is due to a less developed pisiforme than that in spotted hyaena, which must have been causing less efficiency of m. flexor carpi ulnaris as compared to that in the modern and cave hyaenas.

Metacarpalia. Mc III similar to the same bone in the spotted hyaena, though showing somewhat stronger dorso-plantar flattening. Mc V longer than in *C. crocuta*. Tuberosities for mm. interossei slighter and less extended distally than in *C. crocuta* and *C. spelaea*, which testifies to relatively less powerful interosseous muscles. The distal ends of the metacarpal bones in *C. eximia* possess a more narrow and sharp keel for the phalanges of the proximal row.

Astragalus differs from that in the spotted and cave hyaenas by smaller length and greater width of the neck causing the whole bone to be relatively broader than in *C. crocuta* and *C. spelaea*. The area for sustentaculum tali of the calcaneum is elongated, with a broad end, the latter resting mainly on the neck of the bone, whereas in *C. crocuta* this area is divided by a deep groove into two separate facets.

Calcaneum differs from that of *C. crocuta* (and more so from the one in the smaller spotted specimen) by a greater length of the shaft (relatively and absolutely), the length of the anterior region of the bone turned to cuboideum being inferior to that in modern and cave hyaenas; m. gastrocnemius in *C. eximia* was attached to a larger lever, and its contraction must have been more powerful than in modern and cave hyaenas. Articular surface for astragalus continues from the sustentaculum tali extending over the anterior process of the calcaneum, which was evidently connected to the astragalus more firmly than in the spotted, striped and cave hyaenas, in which no such prolongation is observable. The dorso-ventral diameter of the corpus of the bone is greater on account of the enlargement of the dorsal border which became convex (in all the other hyaenas mentioned above this border is concave) [see also Gaudry's plate (1)]. The articular surface for cuboideum lies almost perpendicular to the longitudinal axis of the bone; while in the spotted, striated and cave hyaenas it is situated obliquely, the region of the leg and foot in the latter being apparently less firm than in *C. eximia*.

Naviculare pédis marked by a smaller transverse diameter and greater proximo-distal and antero-posterior diameters than in modern and cave hyaenas. The general structure as well as the shape of the articular surfaces of the distal end are rather «*Ictitherium*-like». The proximal articular surface for the head of the astragalus is more deepened; tuberosities for ligaments on the plantar side are stronger than in *C. crocuta*.

Cuneiforme III. Proximo-distal diameter greater than in *C. crocuta*. There are many differences in the structure of articular surfaces for cuboideum, naviculare, Mt III.

Metatarsale IV is distinguished from the Mt IV in the spotted, and more so in the cave and striated hyaenas, by greater length and slenderness; roughness for m. interosseus (on the plantar side) at the same dis-

tance from the proximal end and of equal length as in *C. crocuta*; the antero-posterior diameter of the proximal end is greater than in *C. crocuta*; facet on the unciform process for Mt strongly concave, and not convex as in *C. crocuta*.

Conclusion

The extremities in *C. eximia* display a number of characters primitive as compared to modern and cave hyaenas. This applies to greater slenderness, more freedom of pronation and supination, lesser force of m. biceps, interosseous muscles in the region of metacarpalia, and so on. A long corpus calcanei, and consequently more powerful action of m. gastrocnemius, coupled with greater stability of the entire tarsal region, and with the length of the metatarsal bones, and the extremities in general, testify to considerable mobility and alertness of *C. eximia*, and possibly even to their adaptation to leaps as against the striated, spotted and cave hyaenas. Furthermore, characters specialized in *C. eximia* (dentition—tendency to reduction of deuterocone in P^4, and to transformation of the tooth into a blade) are primitive in the spotted, striated and cave hyaenas, and vice versa, characters specialized in the latter (greatly shortened hind limbs, the general relative reduction and stoutness of the extremities) are primitive in *C. eximia*, though partly specialized, but in an altogether different direction (long limbs, long heelbone as a lever for the m. gastrocnemius playing an important part in leaps). It is therefore quite possible that with increasing knowledge of the osteology of *C. eximia* this form shall have to be placed in a special subgenus, or, rather, in a genus, more safely than it would be possible on the basis of a study of the dentition alone, as done by Kretzoi (2), who splits the genus *Crocuta* into several new genera. Study on the available teeth of *C. eximia* from Pavlodar as well as the one described by Rjabinin (5) from Teraklia (Paleontol. Institute Collection) shows very great variation of *C. eximia*, particularly of its Chinese representatives [«H. variabilis» (6)]. Evidently variation among extinct hyaenas was not less than among the living hyaenas of to-day, which is but natural if one considers the wide palegeographic range of *C. eximia* all the way from Western Europe to Eastern China.

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ERRATA

Vol. XXI, № 3, Article by W. I. Vernadsky, Member of the Academy, page 128, l. 19 from the top; for «Biochemical Laboratory» read «Biogeochemical Laboratory».

Vol. XXI, № 3, Article by P. Cherenkov, page 119, l. 22 from the top; from the formula «$\frac{1}{k}$» should be excluded.

Vol. XXI, № 4, Article by B. N. Schwanwitsch, p. 182, l. 2 from the top; for «If in the case of stereoscopic devices the method of stereomodels seems to put all the doubts aside, some optical illusion may be possibly thought of» read «If in the case of stereoscopic devices some optical illusion may be possibly thought of, the method of stereomodels seems to put all the doubts aside».

Vol. XXI, № 6, Article by A. Eberzin, pages 312—315; for «Kimmeridgian» read «Cimmerian».

Vol. XXII, № 1, Article by A. Belogurov, pages 53—56; for «Acipenser huso L.» and «Acipenser stellatus Pall.» read «Acipenser stellatus Pallas» and «Huso huso (Linné)» respectively.

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